Journal of Statistical Theory and Practice (2020) 14:3
https://doi.org/10.1007/s42519-019-0068-1

ORIGINAL ARTICLE

On Design and Analysis of Funnel Testing Experiments in Webpage Optimization

Sumin Shen[1] · **Zhiyang Zhang**[1] · **Xinwei Deng**[1]

Abstract
In webpage optimization, funnel testing is a popular technique for studying the effects of webpage features. This study focuses on how to properly design and analyze the funnel testing experiments. To address the problem, we propose two new design construction methods for funnel testing experiments. The proposed methods fully utilize the underlying funnel structures such that the projected designs for different funnels have desirable properties. For the analysis of funnel testing experiments, we consider the penalized regression with the heredity principle to obtain parsimonious and interpretable models for each funnel. Thus, the optimization of finding the optimal settings of webpages can be appropriately achieved. The performance of the proposed design and analysis is illustrated through a few numerical examples.

Keywords Conversion rate · Fractional factorial design · Penalized regression

1 Introduction

Webpage optimization, also known as conversion rate optimization, is a process of designing and executing experimentation for improving specific objectives of webpages' usage, such as customer purchases and subscriptions. For example, an online store looks for how to optimize its website for increasing the completion of customer checkouts. The experimenters often conduct experiments by changing the features of webpages to find which changes will lead to more conversions. Here the *conversion* refers to that visitor have completed the objective of interest. The webpages where conversions (e.g., order placement) can happen are called *conversion points*. The

Part of special issue guest edited by Pritam Ranjan and Min Yang—Algorithms, Analysis and Advanced Methodologies in the Design of Experiments.

✉ Xinwei Deng
xdeng@vt.edu

[1] Department of Statistics, Virginia Tech, Blacksburg, VA 24061, USA

series of webpages that visitors go through until conversion happens is called the *conversion funnel* or simply *funnel*. One favorite webpage optimization technique is the funnel testing, also known as the multi-page testing [2, 11, 23]. It investigates the effects of webpage features on the percentage of visitors completing the objective of interest (denoted as the *conversion rate*) when they go through a series of webpages. A key research question is how to properly design and analyze the funnel testing experiments.

Figure 1 shows an illustrative example of funnel testing in the setting of online shopping. Specifically, visitors arrive on the landing page with A and B as two control factors. The landing page can link to various webpages, where visitors can perform a series of actions such as reading product description, viewing product images, checking product reviews, comparing with similar products. As shown in Fig. 1, (landing page, page 1, page 2) is a funnel, denoted as CF_1, with page 2 as the conversion point. Similarly, (landing page, page 1) is another funnel, denoted as CF_3, with page 1 as the conversion point. And (landing page, page 2) is the third funnel, denoted as CF_2, with page 2 as the conversion point. All the conversion funnels together with associated webpages make up the *conversion system*.

The primary goal of design and analysis of funnel testing experiments is to maximize the total conversion rate. In the literature, research on how to construct the design for funnel testing experiments and optimize its total conversion rate is limited. A straightforward method considers the conversion point where its conversion rate is low, and it becomes the A/B or multivariate testing problem [5]. However, such a method ignores the interactions between different webpages in a funnel. The conversion rate in a funnel is often affected by multiple webpages in the funnel.

Recently, Su and Wu [19] proposed to consider the total conversion rate as a linear combination of conversion rates from the individual funnels. Their framework is

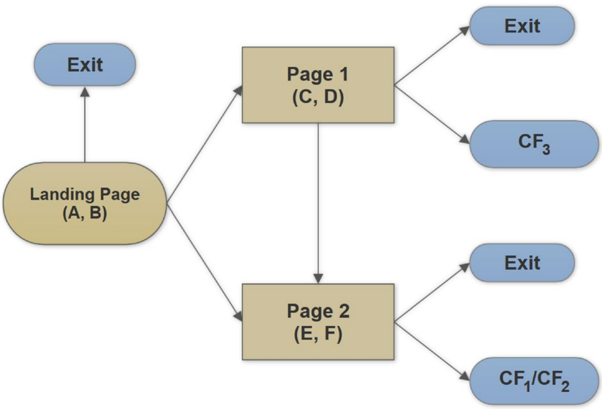

Fig. 1 A toy example of the funnel testing experiment with three funnels. The CF_1 is the conversion funnel from the landing page to page 2 via page 1. The CF_2 is the conversion funnel from the landing page to page 2 directly. The CF_3 is the conversion funnel from the landing page to page 1 directly. Each webpage has two factors: A and B on the landing page, C and D on page 1, and E and F on page 2. Arrows indicate possible visitor's decisions on that webpage. Exit means visitors leave the system on that page

to construct a whole fractional factorial design for the factors from all funnels. The design for a specific funnel is then obtained by projecting the whole design to the factors associated with that funnel. The corresponding analysis of conversion rates is conducted for each funnel. Clearly, constructing a whole design, such as a minimum aberration design, for all factors is conceptually easy, but is practically challenging for a large number of factors at a given design size. Moreover, such a design approach may overlook the underlying structure of the conversion funnels. It could be difficult to justify the construction of the whole design when the conversion funnels have different structures. Lastly, the properties of projected designs for specific conversion funnels may not be desirable.

In this work, we propose two methods of constructing designs for the funnel testing experiments. The proposed construction strategy is to fully utilize the underlying conversion funnel structures such that the projected designs for different funnels have desirable properties. The first method is to augment the design by considering the design for each funnel in a sequential fashion. Such a design can accommodate a large number of funnels and a large number of factors with an economic run size. The second method considers a funnel-driven D-optimal design such that the desirable interaction effects can be accurately estimated. The run size of such D-optimal designs can be quite flexible. For the analysis of the funnel testing experimental data, we consider the penalized regression model with the heredity principle [4, 21]. It enables a parsimonious model fitting with meaningful interpretation and good flexibility for optimization. The proposed design and analysis have several major advantages: First, the proposed methods adapt to the characteristics of funnel testing experiments in terms of different and complex structures of conversion funnels. Second, the proposed designs maintain attractive properties when projecting the design onto the individual conversion funnels. Third, the proposed methods are applicable to the funnel testing experiments with a large number of factors.

Throughout the paper, we consider designs with two levels for the funnel system. We also assume that visitors can exit from any webpage and do not complete conversions on the landing page. The rest of paper is organized as follows. In Sect. 2, we detail the proposed methods to construct designs. In Sects. 3 and 4, we illustrate the proposed methods in the examples. In Sect. 5, we perform a simulation case study. We conclude this work with some discussion in Sect. 6.

2 Design Construction

2.1 Preliminary Setup

Consider a funnel testing experiment with m conversion funnels, denoted as $\mathrm{CF}_1, \dots, \mathrm{CF}_m$. Each funnel has several web pages starting from the landing page and ending at the conversion point. Note that each webpage contains some control factors of interest. Thus, for an individual funnel CF_i, we can write $\mathrm{CF}_i = \{X_j^{(i)} : j = 1, \dots, p_i\}$, where $X_j^{(i)}$ is the jth factor in the funnel CF_i and p_i is the number of factors in the funnel CF_i. Let D denote as the whole design for the funnel system. Then, each funnel has a corresponding design D_i by projecting design D to the corresponding factors for that funnel.

If the funnel CF_i contains all the factors belonging to another funnel $\mathrm{CF}_{i'}$, we denote it as $\mathrm{CF}_{i'} \subseteq \mathrm{CF}_i$. The number of factors contained in the funnel CF_i is defined as the length of the funnel CF_i, denoted as $|\mathrm{CF}_i|$. Without loss of generality, we assume that the funnels are indexed as $|\mathrm{CF}_1| \geq \cdots \geq |\mathrm{CF}_m| > 0$.

For a factor X in the funnel system, it can be characterized into two categories from an experimental design perspective: basis factor, X^{b}, and generator factor, X^{g}. The basis factors are factors used to construct the basic design in the fractional factorial design. The generator factors are the chosen generators in the generating relations of the fractional factorial design [21].

It is worth remarking that in the funnel system, the factors on the landing page are shared by all funnels and thus the shared factors between different funnels always exist.

2.2 Method I

The key idea of the method I is to construct design for the funnel with the largest number of factors (i.e., CF_1) and gradually expand the design to accommodate other funnels (CF_2, CF_3, ...) in a sequential fashion. The shared factors between funnels will have the same columns in the whole design. The proposed method I contains two key procedures: *the partial-expanding procedure* and *the fold-over augmentation procedure*. We will first detail these two procedures.

Figure 2 shows an illustration example how to expand the design to accommodate the second funnel (with factors X_1, X_2, X_6, and X_7), given the design $D^{\dagger}$ of a first funnel (with factors X_1, X_2, X_3, X_4, and X_5). As shown in Fig. 2, the key is on how to construct D^* (red areas) and D^{**} (green areas) such that the design for the second funnel has attractive properties. Here the construction of D^* is obtained by the *partial-expanding procedure*, which consists of three rules: Rule 1, consider a factorial design for $D^{\ddagger}$ (with factors in the second funnel). If unwanted alias structures are generated, then consider a fractional factorial design. Rule 2, if $D^{\ddagger}$ is a fractional factorial design, then both factor types, X^{b} and X^{g}, exist. The shared factors between $D^{\dagger}$ and $D^{\ddagger}$ will have priority to be used as basis factors over generator factors. The factors in D^* are given priority to be used as generator factors over basis factors. Rule 3, if any factor in D^* is to be used as a generator factor in $D^{\ddagger}$, then the corresponding column in D^* is constructed according to the defining relation. For the example of Fig. 2, the design of the first funnel, $D^{\dagger}$, is given by 2_{V}^{5-1} using $5 = 1234$. Then based on Rule 1 and Rule 2, we reject the choice of 2^4 factorial design for $D^{\ddagger}$ because severe alias structures between the factors X_1, X_2, X_3, X_4, X_5, and X_6 are generated. Alternatively, we would consider to construct D^* by using 2_{IV}^{4-1} with the defining relation $7 = 126$.

The *fold-over augmentation procedure* is then used for constructing the augment design D^{**} after the design D^* is constructed. Here we borrow the idea of the fold-over technique [14] and choose the traditional full fold-over plan. That is, we switch the signs of all columns in D^*. Note that it is also possible to use other techniques,

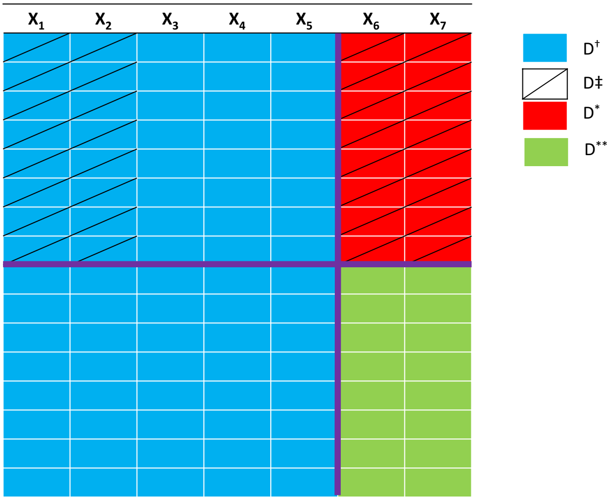

Fig. 2 The illustration example for the partial-expanding procedure and the fold-over augmentation procedure. There are four different designs: $D^\dagger$ (with factors X_1, X_2, X_3, X_4, and X_5), $D^\ddagger$ (with factors X_1, X_2, X_6, and X_7), D^* (with factors X_6 and X_7), and D^{**} (with factors X_6 and X_7). The designs $D^\dagger$ and $D^\ddagger$ share the factors X_1 and X_2. The design size of $D^\dagger$ is equal to that of the combined design of D^* and D^{**}, but larger than that of $D^\ddagger$

such as the fractional factorial design and the D-optimal design, to construct the design D^{**}. There are two advantages of using the fold-over technique: First, the fold-over technique does not create alias structures in the whole design. Second, the fold-over technique does not deteriorate the properties of the design projected from the whole design to the corresponding factors. Note that it is possible that after the fold-over augmentation procedure, the design size of the combined design of D^* and D^{**} is still smaller than that of $D^\dagger$. In that case, we make replicates of the combined design of D^* and D^{**} until the resulting design has a design size equal to that of $D^\dagger$.

For a funnel system with multiple funnels, we can describe the proposed construction method I as follows:

Step 1 Start with the longest funnel CF_1 and build a (fractional) factorial design as its corresponding design D_1. If $|CF_2| = |CF_1|$, then stack the copy of design D_1 above the design D_1, and use the combined design as the new D_1. Set $k = 1$ and design $D = D_1$.

Step 2 Expand the design D by constructing design D_{k+1} for CF_{k+1} when CF_{k+1} contains factors that are not in $CF_1, \ldots, CF_k$. We use similar structures in Fig. 2 with D as the design $D^\dagger$ and D_{k+1} as the design $D^\ddagger$. We construct D^* and D^{**} by the partial-expanding procedure and the fold-over augmentation procedure, respectively.

Step 3 Set $k = k + 1$ and go to **Step 2** until all conversion funnels are accommodated.

Note that the run size of the whole design D is determined by the size of D_1, and it will not change after **Step 1**. Thus the proposed method I can apply to the complex multiple-funnel systems with a large number of factors. Since the proposed method constructs the design by considering the individual funnel sequentially, it can ensure desired properties for the design in each funnel. Moreover, the proposed construction method will not generate serious alias structures in the whole design D because of the fold-over augmentation procedure.

Example 1: Method I for Case 1 We illustrate the method I with a toy example (denote as case 1) similar to the one used in Su and Wu [19]. Figure 1 shows the represented conversion system where there are three funnels: $CF_1 = \{A, B, C, D, E, F\}$, $CF_2 = \{A, B, E, F\}$, and $CF_3 = \{A, B, C, D\}$ with $|CF_1| > |CF_2| = |CF_3|$. CF_1 is the conversion funnel from the landing page to page 2 via page 1. CF_2 is the conversion funnel from the landing page to page 2 directly. Note that page 2 is contained in both funnels CF_1 and CF_2. CF_3 is the conversion funnel from the landing page to page 1 directly. Each page has two factors: A and B on the landing page, C and D on page 1, E and F on page 2. Arrows indicate possible visitor's decisions on that webpage. On the landing page, the three possible decisions are: go to page 1, go to page 2, or exit. On page 1, the three possible decisions are: complete a conversion, go to page 2, and exit. On page 2, the two possible decisions are: complete a conversion or exit.

According to the method I, we start with the longest funnel CF_1. Suppose the design size is limited to 32, then we choose the fractional factorial design 2^{6-1}_{VI} with the design generator $F = ABCDE$. Since $CF_2 \subseteq CF_1$ and $CF_3 \subseteq CF_1$, no more action is needed for expanding the design. Thus the resulting whole design D for case 1 is 2^{6-1}_{VI} with the design generator $F = ABCDE$. The corresponding D_1 is a 2^{6-1}_{VI} with the design generator $F = ABCDE$. D_2 and D_3 are 2^4 factorial designs. Factorial designs are efficient and allow the effects of factors to be estimated while other factors are present [15].

Example 2: Method I for Case 2 We illustrate the method I for another toy example (denote as case 2), which is used in Su and Wu [19]. Figure 3 shows the represented conversion system for case 2, where page 2 has different factors in different funnels CF_1 and CF_3. We denote the variation page as page 2' in the funnel CF_1. This funnel system contains three conversion funnels as $CF_1 = \{A, B, C, D, G, H\}$, $CF_2 = \{A, B, C, D\}$, and $CF_3 = \{A, B, E, F\}$ with $|CF_1| > |CF_2| = |CF_3|$. The other settings of case 2 are the same as case 1. Note that in case 2 each webpage has two factors.

Suppose the design size is limited to 32. According to the method I, we start with the longest funnel CF_1 and choose the fractional factorial design 2^{6-1}_{VI} for D_1 with the design generator $H = ABCDG$. Since $CF_2 \subseteq CF_1$, we will expand the design by constructing D_3 for CF_3 where factors E and F are not contained in D_1. For constructing the design D_3 for CF_3, we can use 2^{6-1}_{VI} as $D^{\dagger}$. By the partial-expanding procedure, we construct a 2^4 factorial design and reverse the signs in D^* to construct D^{**} by the fold-over augmentation procedure.

Table 1 shows the resulting whole design D for case 2. The D_1 is the 2^{6-1}_{VI} with the design generator $H = ABCDG$. The D_2 is a factorial design 2^4 with factors A, B, C, D. The D_3 is the 32-run fold-over design where factors E, F are not aliased with the factors C, D.

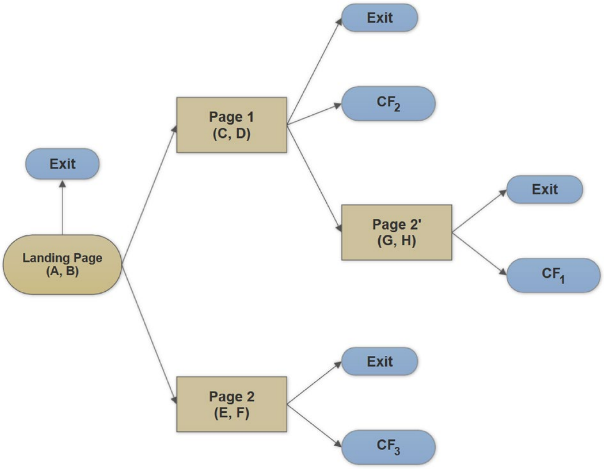

Fig. 3 A second toy example of the funnel testing experiment with three funnels. The CF_1 is the conversion funnel from the landing page to page 2' via page 1. The CF_2 is the conversion funnel from the landing page to page 1 directly. The CF_3 is the conversion funnel from the landing page to page 2 directly. Each webpage has two factors: *A* and *B* on the landing page, *C* and *D* on page 1, *E* and *F* on page 2, and *G* and *H* on page 2'. Arrows indicate possible visitor's decisions on that webpage

Note that when using the Su and Wu [19] method (hereafter called the SW method) for case 2, one would consider a minimum aberration fractional factorial design 2^{8-3}_{IV} with a run size of 32. The design generators of this design are $F = ABC$, $G = ABD$, $H = BCDE$. Then many two-factor interactions are aliased. In our proposed method, the designs for each funnel are either fractional factorial design with resolution V or factorial design. Thus there are no alias structures between two-factor interactions for each funnel. Another possible choice of design based on Su and Wu [19] is a minimum aberration fractional factorial design 2^{8-2}_{V} with a run size of 64. The design generators of this design are $G = ABCD$ and $H = ABEF$. Then the resulting design is two times the size of our proposed design by method I.

2.3 Method II

The method II is to construct a funnel-driven *D*-optimal design for the funnel system. Based on each funnel in the funnel system, we can identify the main effects and two-factor interactions. By considering the main effects and two-factor interactions from all individual funnels, we can construct a *D*-optimal design. Such a design can be more efficient since not all two-factor interactions will be useful given the structures of funnels. For example, for two funnels $CF_1 = \{A, B, C, D, G, H\}$ and $CF_3 = \{A, B, E, F\}$, the two-factor interaction *CE* is not meaningful in the analysis of funnel testing experiments.

Table 1 The 32-run proposed whole design constructed by method I for case 2, and the corresponding simulation data on the leave rate (LR) on the landing page and conversion rates for funnels CF_1, CF_2 and CF_3

	A	*B*	*C*	*D*	*G*	*H*	*E*	*F*	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}
1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	0.1807	0.0000	0.5361	0.0120
2	1	− 1	− 1	− 1	− 1	1	− 1	− 1	0.0764	0.0255	0.3822	0.0127
3	− 1	1	− 1	− 1	− 1	1	− 1	− 1	0.1646	0.0366	0.0366	0.0305
4	1	1	− 1	− 1	− 1	− 1	− 1	− 1	0.1345	0.1228	0.0234	0.0234
5	− 1	− 1	1	− 1	− 1	1	1	− 1	0.1274	0.0000	0.7707	0.0318
6	1	− 1	1	− 1	− 1	− 1	1	− 1	0.1226	0.0000	0.5226	0.0323
7	− 1	1	1	− 1	− 1	− 1	1	− 1	0.1871	0.0129	0.1871	0.0258
8	1	1	1	− 1	− 1	1	1	− 1	0.1083	0.0701	0.1720	0.0255
9	− 1	− 1	− 1	1	− 1	1	− 1	1	0.1657	0.1065	0.3609	0.0473
10	1	− 1	− 1	1	− 1	− 1	− 1	1	0.0963	0.2519	0.2370	0.0074
11	− 1	1	− 1	1	− 1	− 1	− 1	1	0.1149	0.0878	0.2568	0.0338
12	1	1	− 1	1	− 1	1	− 1	1	0.1214	0.2214	0.1429	0.0143
13	− 1	− 1	1	1	− 1	− 1	1	1	0.1064	0.0213	0.5248	0.0284
14	1	− 1	1	1	− 1	1	1	1	0.1192	0.1192	0.3377	0.0132
15	− 1	1	1	1	− 1	1	1	1	0.1385	0.0615	0.3615	0.0462
16	1	1	1	1	− 1	− 1	1	1	0.1421	0.1530	0.3279	0.0219
17	− 1	− 1	− 1	− 1	1	1	1	1	0.1765	0.0000	0.5948	0.0131
18	1	− 1	− 1	− 1	1	− 1	1	1	0.1429	0.0124	0.3602	0.2919
19	− 1	1	− 1	− 1	1	− 1	1	1	0.1529	0.0064	0.0318	0.0446
20	1	1	− 1	− 1	1	1	1	1	0.1046	0.0196	0.0131	0.2680
21	− 1	− 1	1	− 1	1	− 1	− 1	1	0.1768	0.0061	0.6707	0.0549
22	1	− 1	1	− 1	1	1	− 1	1	0.0934	0.0000	0.5055	0.2802
23	− 1	1	1	− 1	1	1	− 1	1	0.2039	0.0000	0.1974	0.0066
24	1	1	1	− 1	1	− 1	− 1	1	0.1027	0.0274	0.2055	0.2534
25	− 1	− 1	− 1	1	1	− 1	1	− 1	0.1233	0.0616	0.3767	0.0548
26	1	− 1	− 1	1	1	1	1	− 1	0.0994	0.1170	0.2281	0.2982
27	− 1	1	− 1	1	1	1	1	− 1	0.1854	0.0464	0.2053	0.0199
28	1	1	− 1	1	1	− 1	1	− 1	0.1607	0.1964	0.1131	0.2738
29	− 1	− 1	1	1	1	1	− 1	− 1	0.0920	0.0491	0.5644	0.0061
30	1	− 1	1	1	1	− 1	− 1	− 1	0.1620	0.0775	0.3310	0.2676
31	− 1	1	1	1	1	− 1	− 1	− 1	0.1925	0.0435	0.4099	0.0124
32	1	1	1	1	1	1	− 1	− 1	0.0927	0.0927	0.3510	0.3113

Here we assume that three-factor and higher interactions are negligible. The *D*-optimal design is to maximize the determinant of information matrix $\mathbf{X}^T\mathbf{X}$ with a fixed number of design points, where $\mathbf{X}$ is the model matrix [9, 10, 15, 22].

We would remark that method II is an algorithmic design, which is known to be flexible in terms of the run size. This is useful when the experimental resources are restricted. However, the resulting alias relations might be more complex than that in the fractional factorial design. In contrast, method I is based on the (fractional) factorial designs, which are orthogonal, balanced, and possessing good projection properties. But method I is not as flexible as method II in the run size.

Example 3: Method II for Case 1 In case 1, there are six main effects and 15 two-factor interaction terms for funnel $CF_1 = \{A, B, C, D, E, F\}$. Since $CF_2 \subseteq CF_1$ and $CF_3 \subseteq CF_1$, the proposed funnel-driven D-optimal design can be constructed based on the model including these six main effects and 15 two-factor interactions. The constructed 32-run D-optimal design is listed in "Appendix 1." The D-efficiency of such a constructed design is 100%.

Example 4: Method II for Case 2 In case 2, there are six main effects and 15 two-factor interactions in $CF_1 = \{A, B, C, D, G, H\}$. Because $CF_2 \subseteq CF_1$, its main effects and two-factor interactions are included in those for CF_1. For $CF_3 = \{A, B, E, F\}$, there are four main effects and six two-factor interactions, but the main effects A and B and the interaction term AB overlap with those in CF_1. By taking the main effects and two-factor interactions from all individual funnels, there are eight main effects and 20 two-factor interactions. Table 2 shows the D-optimal design based on the model accounting for these 28 terms using the package AlgDesign in R software. The D-efficiency of such a constructed design is 91.6%.

3 Analysis of Funnel Testing Experiments

In this section, we consider using the penalized regression with the heredity principle to analyze the data for each funnel with their respective designs. With the fitted models, we can find the optimal settings of factors to maximize the total conversion rate with the consideration of leaving rate on the landing page. For a funnel CF_k, the conversion rate is defined as

$$y_{CF_k} = \frac{\text{number of conversions completed in funnel } CF_k \text{ at a specific design setting}}{\text{number of runs at the specific design setting}}$$

The total conversion rate is computed as a linear combination of conversion rates from each funnel. In this study, we assume equal weights for each conversion funnel. In addition, the leave rate on the landing page is defined as the ratio of the number of visitors left on the landing page and the number of visitors arrived on the landing page.

Specifically, we apply the heredity principle [21] onto the Lasso regression model to produce a more meaningful and interpretable model [4]. The Lasso technique assumes that only a small number of variables are significant [20]. The heredity principle requires that the two-level interaction terms can appear in the model only if one of its main effects appeared in the model [21]. There are two types of heredity principles: weak heredity and strong heredity. The weak heredity principle only requires one of its parent terms to be significant. The strong

Table 2 The 32-run *D*-optimal design constructed by method II for case 2, and the corresponding simulation data on the leave rate (LR) on the landing page and conversion rates for funnels CF_1, CF_2 and CF_3

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}
1	− 1	1	− 1	− 1	− 1	− 1	− 1	− 1	0.1691	0.0294	0.0294	0.0368
2	− 1	− 1	1	− 1	− 1	− 1	− 1	− 1	0.1818	0.0000	0.6818	0.0065
3	1	− 1	− 1	1	1	− 1	− 1	− 1	0.1019	0.1847	0.2803	0.0064
4	− 1	1	− 1	1	1	− 1	− 1	− 1	0.1620	0.1127	0.1972	0.0634
5	1	− 1	− 1	− 1	− 1	1	− 1	− 1	0.1258	0.0629	0.3522	0.0126
6	1	1	1	− 1	− 1	1	− 1	− 1	0.0714	0.0786	0.1214	0.0143
7	− 1	− 1	1	1	1	1	− 1	− 1	0.2023	0.0405	0.4913	0.0231
8	1	1	1	1	1	1	− 1	− 1	0.1522	0.1739	0.2971	0.0145
9	− 1	− 1	− 1	1	− 1	− 1	1	− 1	0.1895	0.0588	0.2941	0.0458
10	1	1	− 1	1	− 1	− 1	1	− 1	0.1097	0.2194	0.1742	0.2258
11	1	− 1	1	− 1	1	− 1	1	− 1	0.1370	0.0000	0.4863	0.2740
12	− 1	1	1	− 1	1	− 1	1	− 1	0.1733	0.0133	0.2467	0.0067
13	1	− 1	1	1	− 1	1	1	− 1	0.1925	0.0559	0.3292	0.2298
14	− 1	1	1	1	− 1	1	1	− 1	0.2159	0.0398	0.3636	0.0682
15	− 1	− 1	− 1	− 1	1	1	1	− 1	0.1608	0.0000	0.4965	0.0210
16	1	1	− 1	− 1	1	1	1	− 1	0.1364	0.0284	0.0227	0.2898
17	1	− 1	1	1	− 1	− 1	− 1	1	0.1623	0.0974	0.3636	0.0195
18	− 1	1	1	1	− 1	− 1	− 1	1	0.1677	0.0659	0.3653	0.0359
19	− 1	− 1	− 1	− 1	1	− 1	− 1	1	0.1644	0.0000	0.5342	0.0479
20	1	1	− 1	− 1	1	− 1	− 1	1	0.1076	0.1203	0.0443	0.0316
21	− 1	− 1	− 1	1	− 1	1	− 1	1	0.1697	0.0788	0.3394	0.0182
22	1	1	− 1	1	− 1	1	− 1	1	0.0921	0.2368	0.1645	0.0066
23	1	− 1	1	− 1	1	1	− 1	1	0.0972	0.0000	0.5347	0.0069
24	− 1	1	1	− 1	1	1	− 1	1	0.1310	0.0179	0.1786	0.0238
25	1	− 1	− 1	− 1	− 1	− 1	1	1	0.0976	0.0183	0.3354	0.2988
26	1	1	1	− 1	− 1	− 1	1	1	0.1071	0.0357	0.1786	0.2321
27	− 1	− 1	1	1	1	− 1	1	1	0.2011	0.0335	0.5028	0.0391
28	1	1	1	1	1	− 1	1	1	0.0921	0.1184	0.3684	0.2171
29	− 1	1	− 1	− 1	− 1	1	1	1	0.1902	0.0109	0.0109	0.0272
30	− 1	− 1	1	− 1	− 1	1	1	1	0.1830	0.0000	0.7059	0.0261
31	1	− 1	− 1	1	1	1	1	1	0.1418	0.1631	0.2270	0.2340
32	− 1	1	− 1	1	1	1	1	1	0.1781	0.0548	0.1644	0.0411

heredity principle selects the interaction term only if both main effects are significant. In this work, we apply the weak heredity principle to the models.

3.1 Simulation on Visitor Actions for Case 2

For illustration, we will focus on case 2 and choose the constructed design from method I as the simulation settings. Following the simulation process in Su and Wu [19] for case 2, we simulate visitor actions to obtain conversion rates for each funnel. Specifically, on the landing page, the visitor first decides to go to page 1 with the probability t_{01}. If the visitor does not go to page 1, then he/she decides to go to page 2 with the conditional probability t_{02}. If the visitor does not go to page 2, then he/she leaves.

Suppose that the visitor is on page 1 from the landing page, he/she first decides to complete a conversion with the probability c_{01}. If the visitor does not complete a conversion on page 1, then he/she decides to go to page 2' with the conditional probability t_{12}. If the visitor does not go to page 2' from page 1, then he/she leaves. If the visitor goes to page 2' from page 1, he/she decides to complete a conversion with the probability c_{12}. If the visitor does not complete a conversion on page 2', then he/she leaves.

Suppose that the visitor is on page 2 from the landing page, he/she decides to complete a conversion with the probability c_{02}. If the visitor does not complete a conversion on page 2, then he/she leaves.

The probability functions are summarized as follows:

$$
\begin{aligned}
&t_{01} = 0.65 - 0.1A,\ t_{02} = 0.52 + 0.21B,\ t_{12} = 0.45 + 0.3A - 0.08AD, \\
&c_{01} = 0.48 - 0.2B + 0.12C + 0.13BD,\ c_{02} = 0.4 + 0.2E + 0.17AE, \\
&c_{12} = 0.47 + 0.29D - 0.1G + 0.02AD + 0.01CG.
\end{aligned}
$$

In the simulation, we perform a randomly replacement sampling for a row from the whole design as the input setting. Each simulation run terminates when the visitor either completes a conversion or leaves the system, and 5000 simulation runs in total were preformed. Table 4 shows the simulated conversion rates for the three conversion funnels and the leave rate (LR) on the landing page when the design is constructed by method I.

3.2 Analysis Results for Case 2

When the Design is Constructed by Method I The fitted models on the conversion rates for the funnels CF_1, CF_2, and CF_3 are:

$$
\begin{aligned}
\hat{y}_{CF_1} &= 0.064 + 0.027A + 0.007B - 0.015C + 0.04D - 0.013G + 0.004AB \\
&\quad - 0.005AC + 0.013AD - 0.006AG - 0.004AH - 0.001BD \\
&\quad - 0.008CD + 0.004CG + 0.003CH - 0.001DG, \\
\hat{y}_{CF_2} &= 0.323 - 0.052A - 0.134B + 0.073C - 0.001D + 0.026AB + 0.067BD, \\
\hat{y}_{CF_3} &= 0.089 + 0.032A.
\end{aligned}
$$

The fitted total conversion rate is

$$\hat{y}_{\mathrm{CF}} = \hat{y}_{\mathrm{CF}_1} + \hat{y}_{\mathrm{CF}_2} + \hat{y}_{\mathrm{CF}_3} = 0.477 + 0.006A - 0.126B + 0.059C + 0.039D - 0.013G + 0.03AB - 0.005AC + 0.013AD - 0.006AG - 0.004AH + 0.066BD - 0.008CD + 0.004CG + 0.003CH - 0.001DG.$$

We fit a linear regression model on the leave rate on the landing page. The fitted y_{LR} is:

$$\hat{y}_{\mathrm{LR}} = 0.136 - 0.019A.$$

Based on the fitted $\hat{y}_{\mathrm{LR}}$, we set A to the +1 level in order to minimize the leave rate on the landing page. Therefore, the optimal settings to maximize the total conversion rate are:

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
+1	−1	−1	−1	*	*	−1	−1

Here the $*$ in the table means that the factors E and F can take either +1 or −1 levels.

When the Design is Constructed by Method II The simulated conversion rates for the three conversion funnels and the leave rate on the landing page are available in Table 2. The fitted models on the conversion rates for the funnels CF_1, CF_2, and CF_3 are:

$$\begin{aligned}
\hat{y}_{\mathrm{CF}_1} &= 0.067 + 0.031A + 0.016B - 0.018C + 0.04D - 0.012G + 0.008AB \\
&\quad - 0.009AC + 0.014AD - 0.004AG - 0.004BG \\
&\quad + -0.01CD + 0.001CG + 0.001GH, \\
\hat{y}_{\mathrm{CF}_2} &= 0.309 - 0.037A - 0.127B + 0.074C + 0.019AB - 0.005AC + 0.063BD, \\
\hat{y}_{\mathrm{CF}_3} &= 0.083 + 0.019A.
\end{aligned}$$

The fitted total conversion rate is

$$\begin{aligned}
\hat{y}_{\mathrm{CF}} = 0.459 + 0.013A - 0.111B + 0.056C + 0.04D - 0.012G + 0.026AB \\
- 0.014AC + 0.014AD - 0.004AG + 0.063BD - 0.004BG \\
- 0.01CD + 0.001CG + 0.001GH.
\end{aligned}$$

The fitted linear regression model on the leave rate on the landing page, y_{LR}, is:

$$\hat{y}_{\mathrm{LR}} = 0.149 - 0.029A.$$

Based on the fitted $\hat{y}_{\mathrm{LR}}$, we set A to the +1 level in order to minimize the leave rate on the landing page. Therefore, the optimal settings to maximize the total conversion rate are:

A	B	C	D	E	F	G	H
+1	−1	−1	+1	*	*	−1	−1

Here the $*$ in the table means that the factors E and F can take either +1 or −1 levels.

When the Design is Constructed by the SW Method Recall that the SW method uses the 32-run fractional factorial design 2_{IV}^{8-3} with design generators as $F = ABC$, $G = ABD$, and $H = BCDE$. The simulated conversion rates for the three conversion funnels and the leave rate on the landing page are listed in "Appendix 2." The fitted models on the conversion rates for the funnels CF_1, CF_2, and CF_3 are:

$$\begin{aligned}
\hat{y}_{\mathrm{CF}_1} &= 0.067 + 0.028A + 0.015B - 0.016C + 0.041D - 0.009G - 0.002H \\
&+ 0.001AB - 0.005AC + 0.013AD - 0.01AG - 0.001BC - 0.004BG \\
&- 0.003BH - 0.005CD + 0.001CG + 0.002CH + 0.003GH, \\
\hat{y}_{\mathrm{CF}_2} &= 0.301 - 0.045A - 0.129B + 0.08C + 0.003D + 0.018AB - 0.017AC \\
&+ 0.004AD - 0.002BC + 0.078BD + 0.01CD, \\
\hat{y}_{\mathrm{CF}_3} &= 0.082.
\end{aligned}$$

It is interesting to observe that the fitted model $\hat{y}_{\mathrm{CF}_3}$ does not contain any significant factor. One possible explanation can be due to the use of Lasso with the heredity principle. The fitted total conversion rate is

$$\begin{aligned}
\hat{y}_{\mathrm{CF}} &= 0.45 - 0.017A - 0.114B + 0.064C + 0.044D - 0.009G \\
&- 0.002H + 0.019AB - 0.022AC + 0.017AD + -0.01AG - 0.004BC \\
&+ 0.078BD - 0.004BG - 0.003BH + 0.005CD \\
&+ 0.001CG + 0.002CH + 0.003GH.
\end{aligned}$$

The fitted linear regression model on the leave rate on the landing page, y_{LR}, is:

$$\hat{y}_{\mathrm{LR}} = 0.155 - 0.032A.$$

Based on the fitted $\hat{y}_{\mathrm{LR}}$, we set A to the +1 level in order to minimize the leave rate on the landing page. Therefore, the optimal settings to maximize the total conversion rate are:

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
+1	−1	−1	−1	*	*	−1	−1

Here the $*$ in the table means that the factors E and F can take either +1 or −1 levels.

It is seen that the method I and the SW method obtain the same optimal settings to maximize the total conversion rate, but the method II obtains a slightly different optimal settings in terms of the factor D. Our proposed methods take into account of the structures in the funnel system, while the SW method mainly focuses all factors in the funnel system. Specifically, the proposed methods I and II consider the potential interaction terms given the structure of the funnel system, while the SW method considers all possible interaction terms, even those that are not meaningful in the context of the funnel system. In the above analyses, the proposed methods can exclude the interaction terms, *CE*, *DE*, *GE*, *HF*, *CF*, *DF*, *GF*, and *HF*, which are unlikely to present in case 2. It is also worth to pointing out that our proposed methods adopt the lasso regression with the heredity principle to obtain sparse and interpretable models, while the SW method in Su and Wu [19] just considered to use the conventional regression for analysis.

In the following example (case 3) in Sect. 4, we further illustrate the importance of taking structures of the funnel system into consideration when constructing the design for the funnel system.

4 Example with Different Number of Factors at Webpages

This case 3, different from the case 2, has different numbers of factors at different webpages. Figure 4 shows the represented conversion system for case 3, where page 1 has three factors C, D, and F; the landing page and the page 2' have two factors; and the page 2 has only one factor, E. This funnel system contains three conversion funnels as $\mathrm{CF}_1 = \{A, B, C, D, F, G, H\}$, $\mathrm{CF}_2 = \{A, B, C, D, F\}$, and $\mathrm{CF}_3 = \{A, B, E\}$ with $|\mathrm{CF}_1| > |\mathrm{CF}_2| > |\mathrm{CF}_3|$. The other settings of case 3 are the same as case 2.

Suppose that the design size is limited to 32. Based on the method I, we start with the longest funnel CF_1 and choose the fractional factorial design 2^{7-2}_{IV} for D_1 with the design generator $G = ABCD$ and $H = ABDE$. Since $\mathrm{CF}_2 \subseteq \mathrm{CF}_1$, we will expand the design by constructing D_3 for CF_3 where the factor E is not contained in D_1. For constructing the design D_3 for D_3, we can use 2^{7-2}_{IV} as $D^{\dagger}$. By the partial-expanding procedure, we construct a repeated 2^3 factorial design of size 16 and reverse the signs in D^* to construct D^{**} by the fold-over augmentation procedure.

Table 3 shows the resulting whole design D for case 3 when the design is constructed by method I. The D_1 is the 2^{7-2}_{IV} with the design generator $G = ABCD$ and $H = ABDE$. The D_2 is a repeated factorial design 2^3 of size 16 with factors A, B, E. The D_3 is the 32-run fold-over design where the factor E is not aliased with the factors C, D, and F.

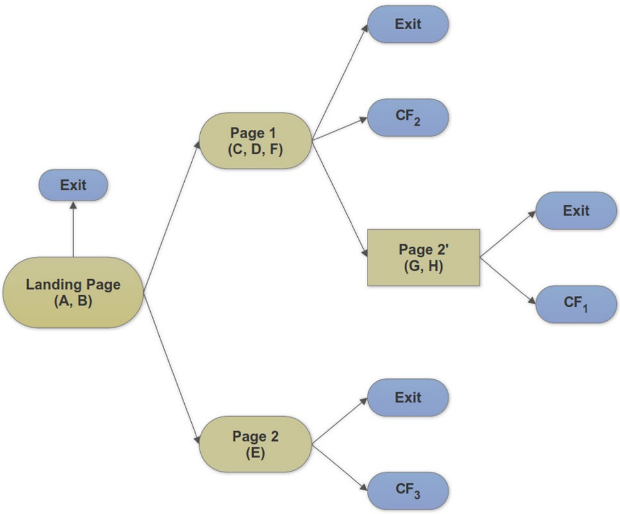

Fig. 4 A third example of the funnel testing experiment with three funnels. The CF_1 is the conversion funnel from the landing page to page 2’ via page 1. The CF_2 is the conversion funnel from the landing page to page 1 directly. The CF_3 is the conversion funnel from the landing page to page 2 directly. Each webpage has different numbers of factors: *A* and *B* on the landing page, *C*, *D*, and *F* on page 1, *E* on page 2, and *G* and *H* on page 2’. Arrows indicate possible visitor’s decisions on that webpage

Note that when using the SW method for case 3, because the total number of factors in case 3 is the same as that in case 2, it will be the same 32-run fractional factorial design 2^{8-3}_{IV} with the design generators $F = ABC, G = ABD, H = BCDG$.

There are seven main effects and 21 two-factor interactions in $\text{CF}_1 = \{A, B, C, D, F, G, H\}$. Because $\text{CF}_2 \subseteq \text{CF}_1$, its main effects and two-factor interactions are included in those for CF_1. For $\text{CF}_3 = \{A, B, E\}$, there are three main effects and three two-factor interactions, but the interaction term AB is overlapped with those in CF_1. By taking the main effects and two-factor interactions from all individual funnels, there are eight main effects and 23 two-factor interactions. Table 4 shows the *D*-optimal design based on the model accounting for these 31 terms using the package AlgDesign in R software. The *D*-efficiency of such a constructed design is 84.5%.

4.1 Analysis Results for Case 3

When the Design is Constructed by Method I The fitted models on the conversion rates for the funnels CF_1, CF_2, and CF_3 are:

$$\begin{aligned}
\hat{y}_{\text{CF}_1} &= 0.061 + 0.026A + 0.005B - 0.013C + 0.041D - 0.014G + 0.004AB \\
&\quad + 0.01AD - 0.007AG - 0.007AH + 0.002BF - 0.013CD - 0.005DG, \\
\hat{y}_{\text{CF}_2} &= 0.315 - 0.033A - 0.128B + 0.073C + 0.003D \\
&\quad + 0.001F - 0.008AC + 0.072BD, \\
\hat{y}_{\text{CF}_3} &= 0.081 + 0.02A.
\end{aligned}$$

Table 3 The 32-run proposed whole design constructed by method I for case 3, and the corresponding simulation data on the leave rate (LR) on the landing page and conversion rates for funnels CF_1, CF_2 and CF_3

	A	*B*	*C*	*D*	*F*	*G*	*H*	*E*	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}
1	− 1	− 1	− 1	− 1	− 1	1	1	− 1	0.1852	0.0000	0.4815	0.0494
2	1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	0.1497	0.0299	0.4072	0.0240
3	− 1	1	− 1	− 1	− 1	− 1	− 1	− 1	0.2109	0.0000	0.0272	0.0136
4	1	1	− 1	− 1	− 1	1	1	− 1	0.0994	0.0292	0.0117	0.2515
5	− 1	− 1	1	− 1	− 1	− 1	1	1	0.2256	0.0000	0.6585	0.0183
6	1	− 1	1	− 1	− 1	1	− 1	1	0.1250	0.0000	0.4940	0.2976
7	− 1	1	1	− 1	− 1	1	− 1	1	0.2014	0.0000	0.2361	0.0486
8	1	1	1	− 1	− 1	− 1	1	1	0.1242	0.0807	0.1180	0.0124
9	− 1	− 1	− 1	1	− 1	− 1	− 1	− 1	0.1871	0.1345	0.2515	0.0175
10	1	− 1	− 1	1	− 1	1	1	− 1	0.1310	0.1517	0.2552	0.2621
11	− 1	1	− 1	1	− 1	1	1	− 1	0.1642	0.0672	0.2537	0.0373
12	1	1	− 1	1	− 1	− 1	− 1	− 1	0.1156	0.2245	0.1361	0.0136
13	− 1	− 1	1	1	− 1	1	− 1	1	0.2115	0.0321	0.4679	0.0256
14	1	− 1	1	1	− 1	− 1	1	1	0.1341	0.1285	0.3799	0.0000
15	− 1	1	1	1	− 1	− 1	1	1	0.1824	0.0189	0.4277	0.0189
16	1	1	1	1	− 1	1	− 1	1	0.0921	0.0855	0.3289	0.2368
17	− 1	− 1	− 1	− 1	1	1	− 1	1	0.2245	0.0000	0.4558	0.0408
18	1	− 1	− 1	− 1	1	− 1	1	1	0.1585	0.0488	0.4573	0.0000
19	− 1	1	− 1	− 1	1	− 1	1	1	0.1687	0.0060	0.0181	0.0120
20	1	1	− 1	− 1	1	1	− 1	1	0.1167	0.0250	0.0167	0.2500
21	− 1	− 1	1	− 1	1	− 1	− 1	− 1	0.1963	0.0000	0.6933	0.0245
22	1	− 1	1	− 1	1	1	1	− 1	0.1410	0.0064	0.5897	0.1859
23	− 1	1	1	− 1	1	1	1	− 1	0.2092	0.0065	0.1830	0.0523
24	1	1	1	− 1	1	− 1	− 1	− 1	0.1448	0.0414	0.1448	0.0069
25	− 1	− 1	− 1	1	1	− 1	1	1	0.1603	0.0641	0.3526	0.0385
26	1	− 1	− 1	1	1	1	− 1	1	0.1329	0.1049	0.2587	0.2657
27	− 1	1	− 1	1	1	1	− 1	1	0.1667	0.0774	0.2381	0.0238
28	1	1	− 1	1	1	− 1	1	1	0.1341	0.2744	0.1524	0.0122
29	− 1	− 1	1	1	1	1	1	− 1	0.2000	0.0129	0.5032	0.0323
30	1	− 1	1	1	1	− 1	− 1	− 1	0.1267	0.1267	0.3533	0.0067
31	− 1	1	1	1	1	− 1	− 1	− 1	0.1013	0.0823	0.4114	0.0316
32	1	1	1	1	1	1	1	− 1	0.1030	0.0909	0.3091	0.2727

The fitted total conversion rate is

$$\begin{aligned}\hat{y}_{CF} = {} & 0.456 + 0.013A - 0.124B + 0.06C + 0.043D + 0.001F - 0.014G \\ & + 0.004AB - 0.008AC + 0.01AD - 0.007AG + 0.006AH \\ & + 0.072BD + 0.002BF - 0.013CD - 0.005DG.\end{aligned}$$

The fitted linear regression model on the leave rate on the landing page, y_{LR}, is:

Table 4 The 32-run proposed whole design constructed by method II for case 3, and the corresponding simulation data on the leave rate (LR) on the landing page and conversion rates for funnels CF_1, CF_2 and CF_3

	A	B	C	D	E	F	G	H	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}
1	− 1	1	1	− 1	− 1	− 1	− 1	− 1	0.1209	0.0055	0.2143	0.0220
2	− 1	− 1	− 1	1	− 1	− 1	− 1	− 1	0.1450	0.0763	0.3817	0.0229
3	1	1	− 1	1	− 1	− 1	− 1	− 1	0.1226	0.2516	0.1161	0.0129
4	1	− 1	− 1	− 1	1	− 1	− 1	− 1	0.1375	0.0438	0.4250	0.0188
5	1	1	1	1	− 1	1	− 1	− 1	0.1471	0.1294	0.3059	0.0000
6	− 1	1	− 1	− 1	1	1	− 1	− 1	0.1778	0.0111	0.0278	0.0222
7	1	− 1	1	− 1	1	1	− 1	− 1	0.1312	0.0062	0.4688	0.0250
8	− 1	− 1	1	1	1	1	− 1	− 1	0.1847	0.0382	0.4968	0.0191
9	1	− 1	1	− 1	− 1	− 1	1	− 1	0.1029	0.0000	0.5294	0.2721
10	− 1	1	− 1	− 1	1	− 1	1	− 1	0.1633	0.0000	0.0204	0.0340
11	− 1	− 1	1	1	1	− 1	1	− 1	0.2027	0.0203	0.5000	0.0270
12	1	1	1	1	1	− 1	1	− 1	0.1090	0.0962	0.3205	0.2949
13	− 1	− 1	− 1	− 1	− 1	1	1	− 1	0.1852	0.0000	0.5494	0.0370
14	1	1	− 1	− 1	− 1	1	1	− 1	0.1314	0.0292	0.0146	0.2336
15	1	− 1	− 1	1	− 1	1	1	− 1	0.0800	0.1667	0.2467	0.2733
16	− 1	1	− 1	1	− 1	1	1	− 1	0.1644	0.0890	0.2397	0.0548
17	− 1	1	1	− 1	1	1	1	− 1	0.2147	0.0061	0.2086	0.0798
18	− 1	1	− 1	− 1	− 1	− 1	− 1	1	0.1987	0.0256	0.0321	0.0321
19	− 1	− 1	1	− 1	1	− 1	− 1	1	0.1728	0.0000	0.6728	0.0247
20	1	1	1	− 1	1	− 1	− 1	1	0.1049	0.0629	0.1119	0.0000
21	1	− 1	1	1	1	− 1	− 1	1	0.0929	0.0643	0.3571	0.0000
22	− 1	1	1	1	1	− 1	− 1	1	0.1707	0.0549	0.4268	0.0244
23	1	− 1	− 1	− 1	− 1	1	− 1	1	0.0964	0.0602	0.4036	0.0120
24	− 1	1	1	− 1	− 1	1	− 1	1	0.1761	0.0189	0.2201	0.0314
25	− 1	− 1	− 1	1	1	1	− 1	1	0.1613	0.0839	0.3161	0.0516
26	1	1	− 1	1	1	1	− 1	1	0.0867	0.1965	0.1329	0.0058
27	− 1	1	1	− 1	− 1	− 1	1	1	0.1657	0.0171	0.2057	0.0286
28	1	− 1	− 1	1	1	− 1	1	1	0.1022	0.1679	0.2482	0.2117
29	− 1	− 1	1	1	− 1	1	1	1	0.1796	0.0120	0.4311	0.0539
30	1	1	1	1	− 1	1	1	1	0.1180	0.1429	0.2733	0.2174
31	− 1	1	− 1	− 1	1	1	1	1	0.1860	0.0116	0.0523	0.0291
32	1	− 1	1	− 1	1	1	1	1	0.0923	0.0000	0.5308	0.2923

$$\hat{y}_{LR} = 0.157 - 0.03A.$$

Based on the fitted $\hat{y}_{LR}$, we set A to the $+1$ level in order to minimize the leave rate on the landing page. Therefore, the optimal settings to maximize the total conversion rate are:

A	B	C	D	E	F	G	H
+1	−1	+1	−1	*	−1	−1	+1

Here the $*$ in the table means that the factor E can take either $+1$ or -1 levels.

When the Design is Constructed by Method II The fitted models on the conversion rates for the funnels CF_1, CF_2, and CF_3 are:

$$\begin{aligned}
\hat{y}_{\mathrm{CF}_1} &= 0.059 + 0.024A + 0.008B + -0.015C + 0.037D - 0.002G + 0.001AB \\
&\quad - 0.004AC + 0.011AD - 0.01CD, \\
\hat{y}_{\mathrm{CF}_2} &= 0.296 - 0.048A - 0.14B + 0.069C + 0.007D - 0.004F + 0.009AB \\
&\quad - 0.005AC - 0.001AD + 0.012BC + 0.083BD \\
&\quad + 0.002BF + 0.002CD - 0.005CF, \\
\hat{y}_{\mathrm{CF}_3} &= 0.077.
\end{aligned}$$

The estimated total conversion rate is

$$\begin{aligned}
\hat{y}_{\mathrm{CF}} &= 0.432 - 0.024A - 0.132B + 0.054C + 0.045D - 0.004F - 0.002G \\
&\quad + 0.009AB + 0.01AD + 0.012BC + 0.083BD + 0.002BF - 0.008CD - 0.005CF.
\end{aligned}$$

The fitted linear regression model on the leave rate on the landing page, y_{LR}, is:

$$\hat{y}_{\mathrm{LR}} = 0.143 - 0.032A.$$

Based on the fitted $\hat{y}_{\mathrm{LR}}$, we set A to the $+1$ level in order to minimize the leave rate on the landing page. Therefore, the optimal settings to maximize the total conversion rate are:

A	B	C	D	E	F	G	H
+1	−1	+1	−1	*	*	−1	*

Here the $*$ in the table means that the factors E, F, and H can take either $+1$ or -1 levels.

When the Design is Constructed by the SW Method Using the SW method, the design for case 3 is the same as that for case 2. Thus, the results will be very similar under the same simulation scheme. However, it is difficult to justify the use of the SW method here since it is expected to have different designs for different funnel system. The structure of the funnel system should play an important role in the construction of designs for the funnel system. Our proposed methods adapt to the structure changes in the funnel systems in cases 2 and 3 and generate different designs. Note that the obtained optimal settings from method I in cases 2 and 3 are different in terms of the factors *C*, *F*, and *H*.

5 Simulation Study for a Complex Funnel System

In the online shopping process, besides the macro-conversion, i.e., product purchases, there are other minor-conversions, such as email subscriptions and customer feedback. Therefore, multiple funnels commonly occur in the conversion system. The number of webpages in each funnel is usually small or moderate. This is because on each webpage, visitors could leave the system. The more webpages in a funnel visitors go through, the more chances for visitors to leave the system.

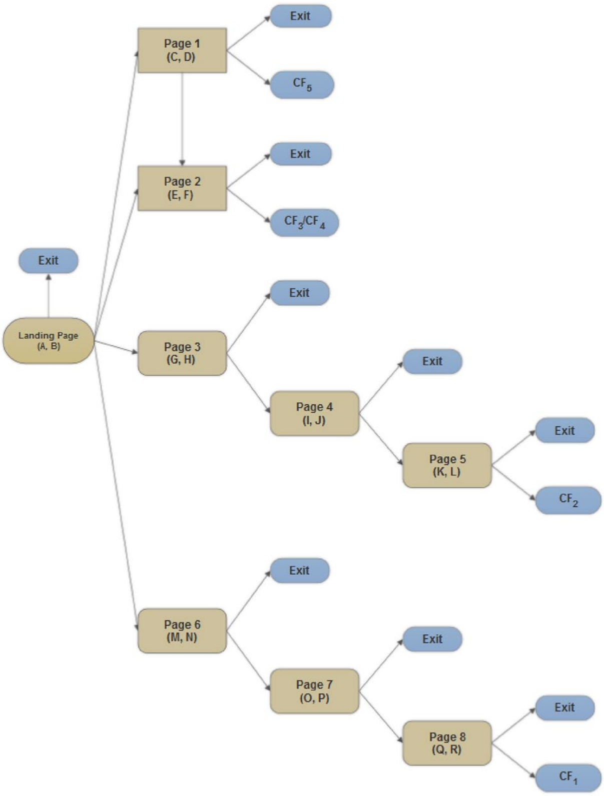

Fig. 5 The simulation study of the funnel testing experiment with five funnels: CF_1, CF_2, CF_3, CF_4, and CF_5. The CF_1 is the conversion funnel from the landing page to page 8 via pages 6 and 7. The CF_2 is the conversion funnel from the landing page to page 5 via pages 3 and 4. The CF_3 is the conversion funnel from the landing page to page 2 via page 1. The CF_4 and CF_5 are the conversion funnels from the landing page to page 2 and page 1, respectively. Each page has two factors denoted by capital alphabet letters

Figure 5 represents the structure of the conversion funnels in the complex funnel system in the simulation study. There are five conversion funnels $CF_1 = \{A, B, M, N, O, P, Q, R\}$, $CF_2 = \{A, B, G, H, I, J, K, L\}$, $CF_3 = \{A, B, C, D, E, F\}$, $CF_4 = \{A, B, E, F\}$, and $CF_5 = \{A, B, C, D\}$. The CF_1 is the conversion funnel from the landing page to page 8 via pages 6 and 7. The CF_2 is the conversion funnel from the landing page to page 5 via pages 3 and 4. The CF_3 is the conversion funnel from the landing page to page 2 via page 1. The CF_4 and CF_5 are the conversion funnels from the landing page to page 2 and page 1, respectively. Each page has two factors. Arrows indicate the possible visitor's decisions on that webpage.

This simulation case contains specific scenarios where a page could vary depending on the linking source page. Page 2 is in both funnels CF_3 and CF_4. We assume that page 2 does not change regardless of the previous webpage the visitor has visited before coming to page 2. This is not the case for page 3. Page 3 has two variants, and we denote page 7 as the other variant if visitors come from page 6 instead of the landing page.

Example 5: Method I for the Simulation Case Using the proposed method I, we start with the longest funnel CF_1 and choose D_1 to be a 64-run fractional factorial design 2_{V}^{8-2} with the design generator $Q = ABMN$ and $R = ABOP$. Since $|CF_2| = |CF_1|$, we follow **Step 1** to stack the copy of the design D_1 on top of D_1 and use the combined design as the new D_1, which is a 128-run design. In **Step 2**, we use D_1 as $D^\dagger$ as demonstrated in Fig. 2. Based on the partial-expanding procedure, we reject the use of the factorial design 2^8 for D_2 due to the large run size. Because of $CF_2 \cap CF_1 = \{A, B\}$ and $CF_2 \setminus CF_1 = \{G, H, I, J, K, L\}$, we propose a fractional factorial design 2_{V}^{8-2} with the design generators $K = ABGH$ and $L = ABIJ$. According to the fold-over augmentation procedure, we reverse the signs of all columns in D^* to construct D^{**}. The resulting design with columns A, B, G, H, I, J, K, and L is the constructed D_2.

In **Step 3**, we use D_2 as $D^\dagger$. Because of $CF_3 \cap CF_2 = \{A, B\}$ and $CF_3 \setminus CF_2 = \{C, D, E, F\}$, we propose a 32-run fractional factorial design 2_{VI}^{6-1} with the design generator $F = ABCDE$. Based on the fold-over augmentation procedure, to construct D^{**} we reverse the signs of all columns in D^*. The size of the combined design of D^* and D^{**} is 64, smaller than the size of D_1. Thus we make a copy of the combined design of D^* and D^{**} to allow the resulting design with a run size of 128. Lastly, since $CF_4 \subseteq CF_3$ and $CF_5 \subseteq CF_3$, there is no need to further expand the design.

The whole resulting design with a run size of 128 is reported in "Appendix 3." The D_1 is the 64-run fractional factorial design 2_{V}^{8-2} with the design generator $Q = ABMN$ and $R = ABOP$. The D_2 is the 128-run design with columns A, B, G, H, I, J, K, and L. The D_3 is the 32-run fractional factorial design 2_{VI}^{6-1} with the design generator $F = ABCDE$. Both D_4 and D_5 are 2^4 factorial designs.

Note that when using the SW method for the simulation study, one would consider a minimum aberration fractional factorial design 2_{IV}^{18-11} with a run size of 128. The design generators of this design are $H = ABCD, I = ABCE, J = ABDF, K = ACEF, L = ADEF, M = ACDG, N = ABEG,$

$O = ADEG, P = ABFG, Q = ACFG, R = BCDEFG$. There are many two-factor interactions are aliased.

Example 6: Method II for the Simulation Case Based on the funnel-driven D-optimal design method, we can identify the main effects and two-factor interactions for each funnel. For $\text{CF}_1 = \{A, B, M, N, O, P, Q, R\}$, $\text{CF}_2 = \{A, B, G, H, I, J, K, L\}$, $\text{CF}_3 = \{A, B, C, D, E, F\}$, there are certain overlapped main effects and two-factor interactions. Since both CF_4 and $\text{CF}_5 \subseteq \text{CF}_3$, their main effects and two-factor interactions are included in those from CF_3. Considering all five funnels, the total number of main and two-factor interaction terms of interest is 87. Note that some interaction terms such as EG and EI are impossible to occur. This is because under the structures of funnels it is impossible for visitors to visit page 2 and page 4 during their actions. The obtained D-optimal design has a D-efficiency of 88.9% and is reported in "Appendix 3."

5.1 Simulation on Visitors' Actions

To simulate visitors' actions for obtaining the conversion rates for the five funnels and the leave rate on the landing page, we consider the scenario as follows. On the landing page, the visitor first decides to go to page 1 with the probability t_{01}. If the visitor does not go to page 1, then he/she decides to go to page 2 with the conditional probability t_{02}. If the visitor does not go to page 2, then he/she decides to go to page 3 with the conditional probability t_{03}. If the visitor does not go to page 3, then he/she decides to go to page 6 with the conditional probability t_{06}. If the visitor does not go to page 6, then he/she leaves.

Suppose that the visitor is on page 1 from the landing page, he/she first decides to complete a conversion with the probability c_{01}. If the visitor does not complete a conversion on page 1, then he/she decides to go to page 2 with the conditional probability t_{12}. If the visitor does not go to page 2 from page 1, then he/she leaves. If the visitor goes to page 2 from page 1, he/she decides to complete a conversion with the probability c_{12}. If the visitor does not complete a conversion on page 2, then he/she leaves.

Suppose that the visitor is on page 2 from the landing page, he/she decides to complete a conversion with the probability c_{02}. If the visitor does not complete a conversion on page 2, then he/she leaves.

Suppose that the visitor is on page 3 from the landing page, he/she first decides to go to page 4 with conditional probability t_{34}. If the visitor does not go to page 4 from page 3, then he/she leaves. If the visitor goes to page 4 from page 3, he/she decides to go to page 5 with the probability t_{45}. If the visitor does not go to page 5, then he/she leaves. If the visitor goes to page 5 from page 4, he/she first decides to complete a conversion with the probability c_{45}. If the visitor does not complete a conversion on page 5, then he/she leaves.

Suppose that the visitor is on page 6 from the landing page, he/she first decides to go to page 7 with the conditional probability t_{67}. If the visitor does not go to page

7 from page 6, then he/she leaves. Suppose that the visitor is on page 7 from page 6, he/she first decides to go to page 8 with the conditional probability t_{78}. If the visitor does not go to page 8 from page 6, then he/she leaves. If the visitor goes to page 8 from page 7, he/she first decides to complete a conversion with the probability c_{78}. If the visitor does not complete a conversion on page 8, then he/she leaves.

The probability functions are summarized as follows:

$$
\begin{aligned}
t_{01} &= 0.25 - 0.1A,\ t_{02} = 0.27 + 0.11B + 0.05AB,\\
t_{03} &= 0.53 - 0.05B,\ t_{06} = 0.7 - 0.05A + 0.15AB,\\
c_{01} &= 0.48 + 0.2C + 0.05A,\ t_{12} = 0.55 + 0.25D,\\
c_{12} &= 0.55 + 0.1E + 0.25EC,\ c_{02} = 0.55 + 0.25F + 0.1A,\\
c_{45} &= 0.65 + 0.15L + 0.1KJ,\ t_{34} = 0.55 + 0.25G,\\
t_{45} &= 0.55 + 0.25I,\ t_{67} = 0.55 + 0.25M,\\
t_{78} &= 0.55 + 0.25O,\ c_{78} = 0.65 + 0.15Q + 0.1OP.
\end{aligned}
$$

5.2 Analysis Results in Simulation

When the Design is Constructed by Method I The simulated conversion rates for the three conversion funnels and the leave rate on the landing page are available in "Appendix 3." The fitted models on the conversion rates for the funnels CF_1, CF_2, CF_3, CF_4, and CF_5 are:

$$
\begin{aligned}
\hat{y}_{\mathrm{CF}_1} &= 0.113 - 0.025A + 0.042C - 0.01AC,\\
\hat{y}_{\mathrm{CF}_2} &= 0.039 - 0.007A,\\
\hat{y}_{\mathrm{CF}_3} &= 0.181 + 0.05A + 0.045B + 0.075F + 0.03AB + 0.005AF\\
&\quad - 0.002BE + 0.019BF + 0.005CF,\\
\hat{y}_{\mathrm{CF}_4} &= 0.047 + 0.006A - 0.012B + 0.019G + 0.002H + 0.018I - 0.001J\\
&\quad + 0.001K + 0.009L - 0.002AB + 0.002AI - 0.002AJ - 0.004BG\\
&\quad - 0.003BI - 0.001BL + 0.001GH + 0.004GI + 0.004GL\\
&\quad + 0.001HI + 0.001HJ + 0.004JK + 0.001KL,\\
\hat{y}_{\mathrm{CF}_5} &= 0.024 + 0.003A + 0.01M + 0.011O + 0.002P + 0.004Q + 0.002AB\\
&\quad + 0.001AM + 0.006MO + 0.002OP + 0.001OQ + 0.001PR.
\end{aligned}
$$

The fitted total conversion rate is

$$
\begin{aligned}
\hat{y}_{\mathrm{CF}} = {} & 0.406 + 0.027A + 0.033B + 0.042C + 0.029AB - 0.01AC + 0.075F\\
& + 0.005AF - 0.002BE + 0.019BF + 0.005CF + 0.019G + 0.002H\\
& + 0.018I - 0.001J + 0.001K + 0.009L + 0.002AI - 0.002AJ\\
& - 0.004BG - 0.003BI - 0.001BL + 0.001GH + 0.004GI + 0.004GL\\
& + 0.001HI + 0.001HJ + 0.004JK + 0.001KL + 0.01M + 0.011O + 0.002P\\
& + 0.004Q + 0.001AM + 0.006MO + 0.002OP + 0.001OQ + 0.001PR.
\end{aligned}
$$

We built a linear regression model on the leave rate y_{LR} on the landing page with the factors A and B. The estimated model for y_{LR} is:

$$\hat{y}_{\mathrm{LR}} = 0.064 + 0.022A - 0.011B - 0.036AB.$$

Based on the fitted $\hat{y}_{\mathrm{LR}}$, we set both A and B to the -1 level in order to minimize the leave rate on the landing page. Therefore, the optimal settings to maximize the fitted total conversion rate are:

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	*N*	*O*	*P*	*Q*	*R*
−1	−1	+1	*	+1	+1	+1	+1	+1	+1	+1	+1	+1	*	+1	+1	+1	+1

Here the $*$ in the table means that the factor D and N can take either $+1$ or -1 levels.

When the Design is Constructed by Method II The simulated conversion rates for the three conversion funnels and the leave rate on the landing page are available in "Appendix 3." The fitted models on the conversion rates for the funnels CF_1, CF_2, CF_3, CF_4, and CF_5 are:

$$\begin{aligned}
\hat{y}_{\mathrm{CF}_1} &= 0.117 - 0.029A + 0.043C - 0.014AC,\\
\hat{y}_{\mathrm{CF}_2} &= 0.037 - 0.007A,\\
\hat{y}_{\mathrm{CF}_3} &= 0.186 + 0.057A + 0.055B + 0.003C + 0.082F + 0.033AB - 0.001AC\\
&\quad - 0.002AE + 0.014AF + 0.022BF + 0.001CF,\\
\hat{y}_{\mathrm{CF}_4} &= 0.043 + 0.002A - 0.006B + 0.016G + 0.02I - 0.001J + 0.009L + 0.001AJ\\
&\quad + 0.006GI + 0.001GL - 0.002HI + 0.003IL + 0.001JK,\\
\hat{y}_{\mathrm{CF}_5} &= 0.025 + 0.006M + 0.01O + 0.004Q - 0.001BQ + 0.003MO.
\end{aligned}$$

The fitted total conversion rate is

$$\begin{aligned}
\hat{y}_{\mathrm{CF}} = 0.408 &+ 0.022A + 0.049B + 0.046C + 0.033AB - 0.015AC + 0.082F\\
&- 0.002AE + 0.014AF + 0.022BF + 0.001CF + 0.016G + 0.02I - 0.001J\\
&+ 0.009L + 0.001AJ + 0.006GI + 0.001GL - 0.002HI + 0.003IL\\
&+ 0.001JK + 0.006M + 0.01O + 0.004Q - 0.001BQ + 0.003MO.
\end{aligned}$$

The fitted linear regression model on the leave rate y_{LR} on the landing page is:

$$\hat{y}_{\mathrm{LR}} = 0.062 + 0.021A - 0.016B - 0.035AB.$$

Based on the fitted $\hat{y}_{\mathrm{LR}}$, we set both A and B to the -1 level in order to minimize the leave rate on the landing page. Therefore, the optimal settings to maximize the fitted total conversion rate is:

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	*N*	*O*	*P*	*Q*	*R*
−1	−1	+1	*	+1	+1	+1	−1	+1	−1	−1	+1	+1	*	+1	*	+1	*

Here the $*$ in the table means that the factor D, N, P, and R can take either +1 or −1 levels.

When the Design is Constructed by the SW Method Since the SW method considers the fractional factorial design based on all factors, one can construct a 128-run 2_{IV}^{18-11} design with the design generator $H = ABCD, I = ABCE, J = ABDF, K = ACEF, L = ADEF, M = ACDG, N = ABEG, O = ADEG, P = ABFG, Q = ACFG$, and $R = BCDEFG$. Clearly, such design would lead to complex aliasing relations among two-factor interactions. The simulated conversion rates for the three conversion funnels and the leave rate on the landing page can be generated accordingly. The fitted models on the conversion rates for the funnels CF_1, CF_2, CF_3, CF_4, and CF_5 are:

$$
\begin{aligned}
\hat{y}_{\text{CF}_1} &= 0.116 - 0.032A + 0.045C - 0.011AC,\\
\hat{y}_{\text{CF}_2} &= 0.039,\\
\hat{y}_{\text{CF}_3} &= 0.187 + 0.051A + 0.048B + 0.078F + 0.033AB + 0.003AF + 0.009BF,\\
\hat{y}_{\text{CF}_4} &= 0.044 + 0.002A - 0.01B + 0.015G + 0.021I + 0.002J + 0.002L - 0.002AB\\
&\quad - 0.001AH + 0.002AI - 0.002BG + 0.001BH\\
&\quad - 0.009BI + 0.007GI + 0.001JK,\\
\hat{y}_{\text{CF}_5} &= 0.024 + 0.007M + 0.009O + 0.003Q + 0.003MO + 0.003MQ\\
&\quad + 0.003OP + 0.001OQ.
\end{aligned}
$$

The fitted total conversion rate is

$$
\begin{aligned}
\hat{y}_{\text{CF}} = {} & 0.409 + 0.021A + 0.038B + 0.045C + 0.031AB - 0.011AC + 0.078F\\
& + 0.003AF + 0.009BF + 0.015G + 0.021I + 0.002J + 0.002L - 0.001AH\\
& + 0.002AI - 0.002BG + 0.001BH - 0.009BI + 0.007GI + 0.001JK + 0.007M\\
& + 0.009O + 0.003Q + 0.003MO + 0.003MQ + 0.003OP + 0.001OQ.
\end{aligned}
$$

The fitted linear regression model on the leave rate y_{LR} on the landing page is:

$$\hat{y}_{\text{LR}} = 0.06 + 0.016A - 0.007B - 0.037AB.$$

Based on the fitted $\hat{y}_{\text{LR}}$, we set both A and B to the −1 level in order to minimize the leave rate on the landing page. Therefore, the optimal settings to maximize the fitted total conversion rate is:

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	*N*	*O*	*P*	*Q*	*R*
−1	−1	+1	*	*	+1	+1	+1	+1	+1	+1	+1	+1	*	+1	+1	+1	*

Here the $*$ in the table means that the factor D, E, N, and R can take either +1 or −1 levels.

The method I and the SW method obtain the same optimal settings to maximize the total conversion rate except the factors *E* and *R*. The method II and the SW method obtain the same optimal settings to maximize the total conversion rate except the factors *E*, *H*, *J*, *K*, and *P*.

6 Discussion

In this work, we propose two new methods to construct designs for the webpage funnel testing experiments. The proposed methods consider the structures of individual funnels in the funnel system and are applicable to the funnel testing experiments with a large number of factors and complex structures of funnels. For the analysis of funnel testing experiments, we consider the penalized regression with the heredity principle to obtain parsimonious and interpretable models for each funnel.

Note that this work mainly considers designs with two-level factors. The proposed methods can be extended to the three-level factorial designs or mixed two-level and three-level factorial designs. For method I, we can use three-level (fractional) factorial designs in Step 1 and in the partial-expanding procedure in Step 2. The fold-over augmentation procedure might not be straightforward to implement under the three-level (fractional) factorial design. An alternative possibility is to consider a three-level *D*-optimal augmentation design [3]. For the mixed-level case, the fold-over augmentation procedure can be even more complex. One possible strategy is replacing three-level factor by two two-level factors [1, 15]. For method II, because of using *D*-optimal design, we can construct a three-level or mixed-level *D*-optimal design directly.

Currently, we index the funnels based on the number of factors in the funnel. It will be an interesting topic in the future to incorporate the number of webpages of the funnel into the design construction procedure. We also would like to point out that the proposed design strategy can be extended to accommodate other design types, for example designs with proportion factors [8, 17, 18], Latin hypercube designs [7, 13, 16], and designs with both qualitative and quantitative factors [6] in the funnel testing experiments. Lastly, Li and Lin [12] studied the optimality of fold-over designs in terms of the aberration criterion for two-level fractional factorial designs. One possible future study is to investigate the optimal fold-over plan in the fold-over augmentation procedure.

Acknowledgements The authors would like to sincerely thank the Associate Editor and reviewers for their insightful comments. We also would like to acknowledge the support from National Science of Foundation of China (NSFC-71531004, NSFC-71822103).

Appendix 1: The Constructed Design for Case 1 in Sect. 2.3

See Table 5.

Table 5 The 32-run *D*-optimal design constructed by method II for case 1

	A	*B*	*C*	*D*	*E*	*F*
1	− 1	− 1	− 1	− 1	− 1	− 1
2	1	1	− 1	− 1	− 1	− 1
3	1	− 1	1	− 1	− 1	− 1
4	− 1	1	1	− 1	− 1	− 1
5	1	− 1	− 1	1	− 1	− 1
6	− 1	1	− 1	1	− 1	− 1
7	− 1	− 1	1	1	− 1	− 1
8	1	1	1	1	− 1	− 1
9	− 1	− 1	− 1	− 1	1	− 1
10	1	1	− 1	− 1	1	− 1
11	1	− 1	1	− 1	1	− 1
12	− 1	1	1	− 1	1	− 1
13	1	− 1	− 1	1	1	− 1
14	− 1	1	− 1	1	1	− 1
15	− 1	− 1	1	1	1	− 1
16	1	1	1	1	1	− 1
17	1	− 1	− 1	− 1	− 1	1
18	− 1	1	− 1	− 1	− 1	1
19	− 1	− 1	1	− 1	− 1	1
20	1	1	1	− 1	− 1	1
21	− 1	− 1	− 1	1	− 1	1
22	1	1	− 1	1	− 1	1
23	1	− 1	1	1	− 1	1
24	− 1	1	1	1	− 1	1
25	1	− 1	− 1	− 1	1	1
26	− 1	1	− 1	− 1	1	1
27	− 1	− 1	1	− 1	1	1
28	1	1	1	− 1	1	1
29	− 1	− 1	− 1	1	1	1
30	1	1	− 1	1	1	1
31	1	− 1	1	1	1	1
32	− 1	1	1	1	1	1

Appendix 2: The Constructed Design and the Corresponding Simulation Data for Case 2 in Sect. 3.2

See Table 6.

Table 6 The 32-run whole design constructed by the SW method for case 2, and the corresponding simulation data on the leave rate (LR) on the landing page and conversion rates for funnels CF_1, CF_2 and CF_3

	A	B	C	D	G	H	E	F	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}
1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	1	0.1689	0.0000	0.5203	0.0068
2	1	− 1	− 1	− 1	− 1	1	1	1	0.1133	0.0467	0.3733	0.0067
3	− 1	1	− 1	− 1	− 1	1	1	− 1	0.1987	0.0128	0.0128	0.0385
4	1	1	− 1	− 1	− 1	− 1	− 1	− 1	0.1484	0.1613	0.0065	0.0065
5	− 1	− 1	1	− 1	− 1	1	− 1	− 1	0.1724	0.0000	0.6897	0.0207
6	1	− 1	1	− 1	− 1	− 1	1	− 1	0.1412	0.0059	0.4353	0.0118
7	− 1	1	1	− 1	− 1	− 1	1	1	0.2185	0.0132	0.1722	0.0265
8	1	1	1	− 1	− 1	1	− 1	1	0.0769	0.0490	0.1119	0.0210
9	− 1	− 1	− 1	1	− 1	− 1	1	− 1	0.2081	0.0671	0.2752	0.0403
10	1	− 1	− 1	1	− 1	1	− 1	− 1	0.1623	0.1753	0.2143	0.0260
11	− 1	1	− 1	1	− 1	1	− 1	1	0.2403	0.0714	0.2013	0.0390
12	1	1	− 1	1	− 1	− 1	1	1	0.1049	0.2778	0.1667	0.0000
13	− 1	− 1	1	1	− 1	1	1	1	0.1677	0.0323	0.5548	0.0129
14	1	− 1	1	1	− 1	− 1	− 1	1	0.0710	0.1124	0.4024	0.0000
15	− 1	1	1	1	− 1	− 1	− 1	− 1	0.1548	0.0839	0.3419	0.0258
16	1	1	1	1	− 1	1	1	− 1	0.1419	0.1554	0.3378	0.0135
17	− 1	− 1	− 1	− 1	1	− 1	− 1	− 1	0.1714	0.0057	0.4743	0.0400
18	1	− 1	− 1	− 1	1	1	1	− 1	0.1218	0.0192	0.3846	0.2628
19	− 1	1	− 1	− 1	1	1	1	1	0.2209	0.0233	0.0174	0.0174
20	1	1	− 1	− 1	1	− 1	− 1	1	0.1019	0.0127	0.0382	0.2739
21	− 1	− 1	1	− 1	1	1	− 1	1	0.1748	0.0000	0.6923	0.0559
22	1	− 1	1	− 1	1	− 1	1	1	0.1208	0.0000	0.4832	0.2483
23	− 1	1	1	− 1	1	− 1	1	− 1	0.1697	0.0182	0.2242	0.0364
24	1	1	1	− 1	1	1	− 1	− 1	0.1523	0.0199	0.1325	0.2715
25	− 1	− 1	− 1	1	1	− 1	1	1	0.1933	0.0600	0.2800	0.0267
26	1	− 1	− 1	1	1	1	− 1	1	0.1667	0.1377	0.2174	0.2101
27	− 1	1	− 1	1	1	1	− 1	− 1	0.1709	0.1139	0.2025	0.0316
28	1	1	− 1	1	1	− 1	1	− 1	0.1024	0.1867	0.1386	0.2590
29	− 1	− 1	1	1	1	1	1	− 1	0.1698	0.0314	0.4654	0.0314
30	1	− 1	1	1	1	− 1	− 1	− 1	0.0844	0.1039	0.3831	0.2857
31	− 1	1	1	1	1	− 1	− 1	1	0.1879	0.0485	0.4424	0.0303
32	1	1	1	1	1	1	1	1	0.1629	0.1067	0.2472	0.2416

Appendix 3: The Constructed Designs and the Corresponding Simulation Data for Examples in Sect. 5

The list below include two tables: Table 7 is for method I. Table 8 is for method II.

Table 7 The proposed whole design constructed by method I for the simulation study, and the corresponding simulation data on the leave rate (LR) on the landing page and conversion rates for funnels CF_1, CF_2, CF_3, CF_4, and CF_5

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	*N*	*O*	*P*	*Q*	*R*	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}	y_{CF_4}	y_{CF_5}
1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	−1	−1	−1	−1	1	1	0.000	0.089	0.067	0.078	0.022	0.011
2	1	−1	−1	−1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	0.162	0.054	0.027	0.162	0.027	0.000
3	−1	1	−1	−1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	0.068	0.102	0.011	0.148	0.000	0.000
4	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	−1	−1	−1	−1	1	1	0.039	0.026	0.026	0.247	0.039	0.000
5	−1	−1	1	−1	−1	1	1	−1	−1	−1	−1	1	1	−1	−1	−1	−1	1	0.027	0.200	0.000	0.253	0.080	0.000
6	1	−1	1	−1	−1	−1	1	−1	−1	−1	1	−1	1	−1	−1	−1	1	−1	0.064	0.077	0.000	0.051	0.026	0.064
7	−1	1	1	−1	−1	−1	1	−1	−1	−1	1	−1	1	−1	−1	−1	1	−1	0.058	0.198	0.012	0.047	0.000	0.023
8	1	1	1	−1	−1	1	1	−1	−1	−1	−1	1	1	−1	−1	−1	−1	1	0.040	0.107	0.000	0.493	0.067	0.027
9	−1	−1	−1	1	−1	1	−1	1	−1	−1	−1	1	−1	1	−1	−1	−1	1	0.011	0.099	0.132	0.110	0.011	0.000
10	1	−1	−1	1	−1	−1	−1	1	−1	−1	1	−1	−1	1	−1	−1	1	−1	0.183	0.024	0.037	0.110	0.000	0.012
11	−1	1	−1	1	−1	−1	−1	1	−1	−1	1	−1	−1	1	−1	−1	1	−1	0.093	0.053	0.107	0.053	0.000	0.000
12	1	1	−1	1	−1	1	−1	1	−1	−1	−1	1	−1	1	−1	−1	−1	1	0.043	0.029	0.043	0.478	0.000	0.014
13	−1	−1	1	1	−1	−1	1	1	−1	−1	1	1	1	1	−1	−1	1	1	0.014	0.159	0.029	0.072	0.058	0.000
14	1	−1	1	1	−1	1	1	1	−1	−1	−1	−1	1	1	−1	−1	−1	−1	0.122	0.073	0.037	0.220	0.085	0.024
15	−1	1	1	1	−1	1	1	1	−1	−1	−1	−1	1	1	−1	−1	−1	−1	0.064	0.167	0.026	0.256	0.038	0.013
16	1	1	1	1	−1	−1	1	1	−1	−1	1	1	1	1	−1	−1	1	1	0.037	0.074	0.012	0.235	0.025	0.012
17	−1	−1	−1	−1	1	1	−1	−1	1	−1	1	−1	−1	−1	1	−1	1	−1	0.000	0.153	0.017	0.153	0.000	0.017
18	1	−1	−1	−1	1	−1	−1	−1	1	−1	−1	1	−1	−1	1	−1	−1	1	0.205	0.051	0.013	0.115	0.064	0.000
19	−1	1	−1	−1	1	−1	−1	−1	1	−1	−1	1	−1	−1	1	−1	−1	1	0.043	0.100	0.014	0.057	0.014	0.000
20	1	1	−1	−1	1	1	−1	−1	1	−1	1	−1	−1	−1	1	−1	1	−1	0.044	0.103	0.015	0.338	0.029	0.029
21	−1	−1	1	−1	1	−1	1	−1	1	−1	−1	−1	1	−1	1	−1	−1	−1	0.000	0.169	0.026	0.039	0.065	0.091
22	1	−1	1	−1	1	1	1	−1	1	−1	1	1	1	−1	1	−1	1	1	0.162	0.100	0.000	0.225	0.175	0.050
23	−1	1	1	−1	1	1	1	−1	1	−1	1	1	1	−1	1	−1	1	1	0.099	0.235	0.049	0.235	0.062	0.012
24	1	1	1	−1	1	−1	1	−1	1	−1	−1	−1	1	−1	1	−1	−1	−1	0.032	0.113	0.000	0.129	0.097	0.081
25	−1	−1	−1	1	1	−1	−1	1	1	−1	−1	−1	−1	1	1	−1	−1	−1	0.014	0.068	0.149	0.054	0.027	0.000

Table 7 (continued)

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	*N*	*O*	*P*	*Q*	*R*	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}	y_{CF_4}	y_{CF_5}
26	1	− 1	− 1	1	1	1	− 1	1	1	− 1	1	1	− 1	1	1	− 1	1	1	0.123	0.086	0.025	0.185	0.062	0.000
27	− 1	1	− 1	1	1	1	− 1	1	1	− 1	1	1	− 1	1	1	− 1	1	1	0.055	0.088	0.088	0.264	0.022	0.011
28	1	1	− 1	1	1	− 1	− 1	1	1	− 1	− 1	− 1	− 1	1	1	− 1	− 1	− 1	0.057	0.014	0.071	0.243	0.029	0.014
29	− 1	− 1	1	1	1	1	1	1	1	− 1	1	− 1	1	1	1	− 1	1	− 1	0.024	0.282	0.035	0.188	0.071	0.071
30	1	− 1	1	1	1	− 1	1	1	1	− 1	− 1	1	1	1	1	− 1	− 1	1	0.123	0.178	0.000	0.096	0.178	0.041
31	− 1	1	1	1	1	− 1	1	1	1	− 1	− 1	1	1	1	1	− 1	− 1	1	0.081	0.258	0.097	0.065	0.081	0.032
32	1	1	1	1	1	1	1	1	1	− 1	1	− 1	1	1	1	− 1	1	− 1	0.011	0.068	0.023	0.500	0.045	0.057
33	− 1	− 1	1	1	1	1	− 1	− 1	− 1	1	1	− 1	− 1	− 1	− 1	1	1	− 1	0.022	0.215	0.054	0.215	0.011	0.000
34	1	− 1	1	1	1	− 1	− 1	− 1	− 1	1	− 1	1	− 1	− 1	− 1	1	− 1	1	0.095	0.159	0.000	0.063	0.016	0.000
35	− 1	1	1	1	1	− 1	− 1	− 1	− 1	1	− 1	1	− 1	− 1	− 1	1	− 1	1	0.088	0.088	0.044	0.059	0.000	0.015
36	1	1	1	1	1	1	− 1	− 1	− 1	1	1	− 1	− 1	− 1	− 1	1	1	− 1	0.051	0.165	0.038	0.443	0.000	0.038
37	− 1	− 1	− 1	1	1	− 1	1	− 1	− 1	1	− 1	− 1	1	− 1	− 1	1	− 1	− 1	0.014	0.014	0.137	0.082	0.014	0.000
38	1	− 1	− 1	1	1	1	1	− 1	− 1	1	1	1	1	− 1	− 1	1	1	1	0.123	0.055	0.027	0.151	0.096	0.027
39	− 1	1	− 1	1	1	1	1	− 1	− 1	1	1	1	1	− 1	− 1	1	1	1	0.037	0.024	0.110	0.195	0.024	0.024
40	1	1	− 1	1	1	− 1	1	− 1	− 1	1	− 1	− 1	1	− 1	− 1	1	− 1	− 1	0.071	0.036	0.024	0.190	0.012	0.036
41	− 1	− 1	1	− 1	1	− 1	− 1	1	− 1	1	− 1	− 1	− 1	1	− 1	1	− 1	− 1	0.027	0.203	0.054	0.068	0.000	0.014
42	1	− 1	1	− 1	1	1	− 1	1	− 1	1	1	1	− 1	1	− 1	1	1	1	0.167	0.094	0.010	0.156	0.010	0.010
43	− 1	1	1	− 1	1	1	− 1	1	− 1	1	1	1	− 1	1	− 1	1	1	1	0.061	0.183	0.012	0.220	0.037	0.000
44	1	1	1	− 1	1	− 1	− 1	1	− 1	1	− 1	− 1	− 1	1	− 1	1	− 1	− 1	0.038	0.192	0.000	0.167	0.000	0.000
45	− 1	− 1	− 1	− 1	1	1	1	1	− 1	1	1	− 1	1	1	− 1	1	1	− 1	0.013	0.103	0.026	0.244	0.051	0.026
46	1	− 1	− 1	− 1	1	− 1	1	1	− 1	1	− 1	1	1	1	− 1	1	− 1	1	0.167	0.038	0.000	0.103	0.051	0.026
47	− 1	1	− 1	− 1	1	− 1	1	1	− 1	1	− 1	1	1	1	− 1	1	− 1	1	0.056	0.056	0.069	0.083	0.069	0.000
48	1	1	− 1	− 1	1	1	1	1	− 1	1	1	− 1	1	1	− 1	1	1	− 1	0.013	0.076	0.025	0.468	0.038	0.013
49	− 1	− 1	1	1	− 1	− 1	− 1	− 1	1	1	1	1	− 1	− 1	1	1	1	1	0.000	0.235	0.000	0.015	0.059	0.000

Table 7 (continued)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}	y_{CF_4}	y_{CF_5}
50	1	− 1	1	1	− 1	1	− 1	− 1	1	1	− 1	− 1	− 1	− 1	1	1	− 1	− 1	0.122	0.085	0.000	0.232	0.024	0.000
51	− 1	1	1	1	− 1	1	− 1	− 1	1	1	− 1	− 1	− 1	− 1	1	1	− 1	− 1	0.014	0.211	0.014	0.268	0.014	0.028
52	1	1	1	1	− 1	− 1	− 1	− 1	1	1	1	1	− 1	− 1	1	1	1	1	0.013	0.107	0.000	0.213	0.027	0.053
53	− 1	− 1	− 1	1	− 1	1	1	− 1	1	1	− 1	1	1	− 1	1	1	− 1	1	0.028	0.097	0.139	0.111	0.097	0.042
54	1	− 1	− 1	1	− 1	− 1	1	− 1	1	1	1	− 1	1	− 1	1	1	1	− 1	0.121	0.071	0.051	0.141	0.071	0.061
55	− 1	1	− 1	1	− 1	− 1	1	− 1	1	1	1	− 1	1	− 1	1	1	1	− 1	0.015	0.108	0.231	0.062	0.046	0.015
56	1	1	− 1	1	− 1	1	1	− 1	1	1	− 1	1	1	− 1	1	1	− 1	1	0.014	0.042	0.070	0.408	0.085	0.056
57	− 1	− 1	1	− 1	− 1	1	− 1	1	1	1	− 1	1	− 1	1	1	1	− 1	1	0.032	0.200	0.011	0.189	0.042	0.032
58	1	− 1	1	− 1	− 1	− 1	− 1	1	1	1	1	− 1	− 1	1	1	1	1	− 1	0.059	0.153	0.024	0.047	0.082	0.024
59	− 1	1	1	− 1	− 1	− 1	− 1	1	1	1	1	− 1	− 1	1	1	1	1	− 1	0.082	0.192	0.027	0.027	0.014	0.027
60	1	1	1	− 1	− 1	1	− 1	1	1	1	− 1	1	− 1	1	1	1	− 1	1	0.014	0.137	0.000	0.425	0.014	0.014
61	− 1	− 1	− 1	− 1	− 1	− 1	1	1	1	1	1	1	1	1	1	1	1	1	0.000	0.071	0.029	0.029	0.286	0.086
62	1	− 1	− 1	− 1	− 1	1	1	1	1	1	− 1	− 1	1	1	1	1	− 1	− 1	0.222	0.086	0.000	0.099	0.123	0.012
63	− 1	1	− 1	− 1	− 1	1	1	1	1	1	− 1	− 1	1	1	1	1	− 1	− 1	0.031	0.083	0.052	0.240	0.052	0.031
64	1	1	− 1	− 1	− 1	− 1	1	1	1	1	1	1	1	1	1	1	1	1	0.042	0.083	0.000	0.139	0.139	0.097
65	− 1	− 1	− 1	− 1	− 1	− 1	1	1	1	1	− 1	− 1	− 1	− 1	− 1	− 1	1	1	0.015	0.077	0.046	0.031	0.077	0.031
66	1	− 1	− 1	− 1	− 1	1	1	1	1	1	1	1	− 1	− 1	− 1	− 1	− 1	− 1	0.198	0.033	0.011	0.198	0.220	0.000
67	− 1	1	− 1	− 1	− 1	1	1	1	1	1	1	1	− 1	− 1	− 1	− 1	− 1	− 1	0.074	0.086	0.025	0.222	0.111	0.000
68	1	1	− 1	− 1	− 1	− 1	1	1	1	1	− 1	− 1	− 1	− 1	− 1	− 1	1	1	0.038	0.062	0.038	0.225	0.025	0.000
69	− 1	− 1	1	− 1	− 1	1	− 1	1	1	1	1	− 1	1	− 1	− 1	− 1	− 1	1	0.025	0.253	0.000	0.139	0.013	0.025
70	1	− 1	1	− 1	− 1	− 1	− 1	1	1	1	− 1	1	1	− 1	− 1	− 1	1	− 1	0.138	0.103	0.011	0.092	0.069	0.023
71	− 1	1	1	− 1	− 1	− 1	− 1	1	1	1	− 1	1	1	− 1	− 1	− 1	1	− 1	0.055	0.192	0.014	0.041	0.027	0.000
72	1	1	1	− 1	− 1	1	− 1	1	1	1	1	− 1	1	− 1	− 1	− 1	− 1	1	0.040	0.133	0.000	0.413	0.053	0.000
73	− 1	− 1	− 1	1	− 1	1	1	− 1	1	1	1	− 1	− 1	1	− 1	− 1	− 1	1	0.015	0.092	0.138	0.123	0.062	0.000

Table 7 (continued)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}	y_{CF_4}	y_{CF_5}
74	1	− 1	− 1	1	− 1	− 1	1	− 1	1	1	− 1	1	− 1	1	− 1	− 1	1	− 1	0.120	0.076	0.033	0.152	0.120	0.022
75	− 1	1	− 1	1	− 1	− 1	1	− 1	1	1	− 1	1	− 1	1	− 1	− 1	1	− 1	0.082	0.068	0.151	0.068	0.014	0.000
76	1	1	− 1	1	− 1	1	1	− 1	1	1	1	− 1	− 1	1	− 1	− 1	− 1	1	0.034	0.034	0.000	0.511	0.091	0.000
77	− 1	− 1	1	1	− 1	− 1	− 1	− 1	1	1	− 1	− 1	1	1	− 1	− 1	1	1	0.013	0.280	0.027	0.067	0.040	0.013
78	1	− 1	1	1	− 1	1	− 1	− 1	1	1	1	1	1	1	− 1	− 1	− 1	− 1	0.139	0.125	0.000	0.167	0.069	0.000
79	− 1	1	1	1	− 1	1	− 1	− 1	1	1	1	1	1	1	− 1	− 1	− 1	− 1	0.041	0.149	0.014	0.203	0.041	0.014
80	1	1	1	1	− 1	− 1	− 1	− 1	1	1	− 1	− 1	1	1	− 1	− 1	1	1	0.037	0.085	0.000	0.232	0.024	0.049
81	− 1	− 1	− 1	− 1	1	1	1	1	− 1	1	− 1	1	− 1	− 1	1	− 1	1	− 1	0.027	0.068	0.055	0.260	0.041	0.000
82	1	− 1	− 1	− 1	1	− 1	1	1	− 1	1	1	− 1	− 1	− 1	1	− 1	− 1	1	0.085	0.017	0.000	0.169	0.051	0.017
83	− 1	1	− 1	− 1	1	− 1	1	1	− 1	1	1	− 1	− 1	− 1	1	− 1	− 1	1	0.080	0.057	0.023	0.080	0.023	0.011
84	1	1	− 1	− 1	1	1	1	1	− 1	1	− 1	1	− 1	− 1	1	− 1	1	− 1	0.011	0.011	0.011	0.575	0.023	0.023
85	− 1	− 1	1	− 1	1	− 1	− 1	1	− 1	1	1	1	1	− 1	1	− 1	− 1	− 1	0.038	0.139	0.051	0.051	0.000	0.013
86	1	− 1	1	− 1	1	1	− 1	1	− 1	1	− 1	− 1	1	− 1	1	− 1	1	1	0.107	0.133	0.027	0.280	0.013	0.080
87	− 1	1	1	− 1	1	1	− 1	1	− 1	1	− 1	− 1	1	− 1	1	− 1	1	1	0.085	0.268	0.024	0.171	0.000	0.037
88	1	1	1	− 1	1	− 1	− 1	1	− 1	1	1	1	1	− 1	1	− 1	− 1	− 1	0.051	0.139	0.013	0.152	0.000	0.038
89	− 1	− 1	− 1	1	1	− 1	1	− 1	− 1	1	1	1	− 1	1	1	− 1	− 1	− 1	0.011	0.090	0.045	0.090	0.079	0.000
90	1	− 1	− 1	1	1	1	1	− 1	− 1	1	− 1	− 1	− 1	1	1	− 1	1	1	0.169	0.060	0.012	0.217	0.024	0.012
91	− 1	1	− 1	1	1	1	1	− 1	− 1	1	− 1	− 1	− 1	1	1	− 1	1	1	0.130	0.052	0.078	0.169	0.000	0.039
92	1	1	− 1	1	1	− 1	1	− 1	− 1	1	1	1	− 1	1	1	− 1	− 1	− 1	0.068	0.011	0.034	0.170	0.057	0.034
93	− 1	− 1	1	1	1	1	− 1	− 1	− 1	1	− 1	1	1	1	1	− 1	1	− 1	0.015	0.265	0.118	0.147	0.000	0.044
94	1	− 1	1	1	1	− 1	− 1	− 1	− 1	1	1	− 1	1	1	1	− 1	− 1	1	0.167	0.167	0.030	0.152	0.015	0.000
95	− 1	1	1	1	1	− 1	− 1	− 1	− 1	1	1	− 1	1	1	1	− 1	− 1	1	0.020	0.214	0.102	0.031	0.000	0.041
96	1	1	1	1	1	1	− 1	− 1	− 1	1	− 1	1	1	1	1	− 1	1	− 1	0.034	0.067	0.067	0.416	0.000	0.090
97	− 1	− 1	1	1	1	1	1	1	1	− 1	− 1	1	− 1	− 1	− 1	1	1	− 1	0.024	0.229	0.072	0.169	0.120	0.000

Table 7 (continued)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}	y_{CF_4}	y_{CF_5}
98	1	−1	1	1	1	−1	1	1	1	−1	1	−1	−1	−1	−1	1	−1	1	0.183	0.110	0.049	0.073	0.134	0.000
99	−1	1	1	1	1	−1	1	1	1	−1	1	−1	−1	−1	−1	1	−1	1	0.092	0.246	0.077	0.077	0.031	0.000
100	1	1	1	1	1	1	1	1	1	−1	−1	1	−1	−1	−1	1	1	−1	0.035	0.105	0.012	0.430	0.070	0.012
101	−1	−1	−1	1	1	−1	−1	1	1	−1	1	1	1	−1	−1	1	−1	−1	0.012	0.084	0.084	0.084	0.012	0.000
102	1	−1	−1	1	1	1	−1	1	1	−1	−1	−1	1	−1	−1	1	1	1	0.108	0.041	0.041	0.149	0.122	0.000
103	−1	1	−1	1	1	1	−1	1	1	−1	−1	−1	1	−1	−1	1	1	1	0.058	0.081	0.128	0.221	0.012	0.012
104	1	1	−1	1	1	−1	−1	1	1	−1	1	1	1	−1	−1	1	−1	−1	0.035	0.047	0.047	0.141	0.047	0.024
105	−1	−1	1	−1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	−1	0.014	0.216	0.068	0.041	0.041	0.000
106	1	−1	1	−1	1	1	1	−1	1	−1	−1	−1	−1	1	−1	1	1	1	0.120	0.072	0.000	0.229	0.133	0.012
107	−1	1	1	−1	1	1	1	−1	1	−1	−1	−1	−1	1	−1	1	1	1	0.070	0.291	0.012	0.221	0.070	0.012
108	1	1	1	−1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	−1	0.064	0.141	0.000	0.218	0.064	0.013
109	−1	−1	−1	−1	1	1	−1	−1	1	−1	−1	1	1	1	−1	1	1	−1	0.034	0.067	0.022	0.124	0.090	0.022
110	1	−1	−1	−1	1	−1	−1	−1	1	−1	1	−1	1	1	−1	1	−1	1	0.073	0.061	0.012	0.073	0.049	0.000
111	−1	1	−1	−1	1	−1	−1	−1	1	−1	1	−1	1	1	−1	1	−1	1	0.074	0.086	0.037	0.086	0.025	0.000
112	1	1	−1	−1	1	1	−1	−1	1	−1	−1	1	1	1	−1	1	1	−1	0.085	0.099	0.014	0.366	0.056	0.014
113	−1	−1	1	1	−1	−1	1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	0.011	0.239	0.011	0.011	0.023	0.045
114	1	−1	1	1	−1	1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1	0.083	0.056	0.028	0.278	0.083	0.000
115	−1	1	1	1	−1	1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1	0.057	0.159	0.011	0.250	0.023	0.011
116	1	1	1	1	−1	−1	1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	0.028	0.097	0.000	0.167	0.028	0.056
117	−1	−1	−1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1	1	0.045	0.068	0.125	0.148	0.023	0.057
118	1	−1	−1	1	−1	−1	−1	1	−1	−1	−1	1	1	−1	1	1	1	−1	0.090	0.051	0.090	0.077	0.051	0.128
119	−1	1	−1	1	−1	−1	−1	1	−1	−1	−1	1	1	−1	1	1	1	−1	0.087	0.101	0.101	0.101	0.058	0.072
120	1	1	−1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1	1	0.099	0.000	0.099	0.408	0.000	0.085
121	−1	−1	1	−1	−1	1	1	−1	−1	−1	1	−1	−1	1	1	1	−1	1	0.013	0.184	0.000	0.171	0.013	0.000

Table 7 (continued)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}	y_{CF_4}	y_{CF_5}
122	1	− 1	1	− 1	− 1	− 1	1	− 1	− 1	− 1	− 1	1	− 1	1	1	1	1	− 1	0.171	0.134	0.000	0.085	0.122	0.000
123	− 1	1	1	− 1	− 1	− 1	1	− 1	− 1	− 1	− 1	1	− 1	1	1	1	1	− 1	0.104	0.224	0.015	0.075	0.015	0.030
124	1	1	1	− 1	− 1	1	1	− 1	− 1	− 1	1	− 1	− 1	1	1	1	− 1	1	0.028	0.070	0.000	0.563	0.014	0.028
125	− 1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	1	1	1	1	1	1	0.029	0.116	0.058	0.014	0.014	0.101
126	1	− 1	− 1	− 1	− 1	1	− 1	− 1	− 1	− 1	1	1	1	1	1	1	− 1	− 1	0.107	0.080	0.040	0.293	0.027	0.067
127	− 1	1	− 1	− 1	− 1	1	− 1	− 1	− 1	− 1	1	1	1	1	1	1	− 1	− 1	0.095	0.041	0.122	0.230	0.000	0.041
128	1	1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	1	1	1	1	1	1	0.014	0.056	0.014	0.181	0.000	0.111

Table 8 The proposed whole design constructed by method II for the simulation study, and the corresponding simulation data on the leave rate (LR) on the landing page and conversion rates for funnels CF_1, CF_2, CF_3, CF_4, and CF_5

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	*N*	*O*	*P*	*Q*	*R*	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}	y_{CF_4}	y_{CF_5}
1	1	−1	−1	1	1	−1	−1	−1	−1	−1	−1	1	−1	−1	−1	−1	−1	−1	0.135	0.068	0.041	0.068	0.014	0.000
2	1	1	1	1	−1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	0.046	0.069	0.000	0.494	0.092	0.000
3	−1	1	−1	−1	−1	−1	−1	1	1	−1	1	1	−1	−1	−1	−1	−1	−1	0.052	0.104	0.052	0.065	0.026	0.026
4	−1	1	1	1	−1	−1	−1	1	−1	1	1	−1	1	−1	−1	−1	−1	−1	0.071	0.286	0.036	0.060	0.000	0.000
5	−1	−1	−1	1	−1	−1	1	−1	−1	1	1	1	1	−1	−1	−1	−1	−1	0.012	0.084	0.193	0.060	0.048	0.012
6	−1	1	1	−1	1	1	1	−1	1	1	−1	−1	−1	1	−1	−1	−1	−1	0.038	0.179	0.038	0.269	0.077	0.000
7	1	1	1	1	−1	−1	−1	−1	−1	−1	1	−1	−1	1	−1	−1	−1	−1	0.000	0.068	0.000	0.192	0.000	0.014
8	−1	−1	1	1	1	1	1	1	−1	−1	1	−1	−1	1	−1	−1	−1	−1	0.071	0.229	0.057	0.171	0.029	0.000
9	1	−1	1	1	1	−1	1	−1	1	−1	−1	−1	1	1	−1	−1	−1	−1	0.099	0.127	0.000	0.113	0.085	0.014
10	−1	1	−1	1	1	1	−1	1	−1	−1	−1	1	1	1	−1	−1	−1	−1	0.024	0.061	0.122	0.220	0.000	0.000
11	1	1	1	−1	1	−1	1	−1	−1	1	1	−1	−1	−1	1	−1	−1	−1	0.039	0.091	0.013	0.195	0.013	0.013
12	−1	−1	1	1	−1	1	1	−1	1	1	−1	1	−1	−1	1	−1	−1	−1	0.000	0.217	0.033	0.150	0.100	0.017
13	−1	1	−1	−1	1	1	−1	−1	1	−1	−1	−1	1	−1	1	−1	−1	−1	0.099	0.062	0.012	0.272	0.049	0.012
14	−1	−1	1	−1	1	−1	1	1	−1	1	−1	−1	1	−1	1	−1	−1	−1	0.012	0.288	0.050	0.012	0.012	0.050
15	1	1	1	−1	−1	1	−1	−1	−1	1	1	1	1	−1	1	−1	−1	−1	0.031	0.104	0.000	0.490	0.010	0.031
16	1	−1	−1	−1	−1	1	1	1	−1	−1	1	1	−1	1	1	−1	−1	−1	0.134	0.031	0.010	0.227	0.072	0.021
17	−1	−1	−1	1	1	1	−1	−1	−1	1	1	−1	1	1	1	−1	−1	−1	0.013	0.089	0.089	0.152	0.000	0.051
18	−1	−1	−1	−1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	1	−1	−1	0.025	0.025	0.025	0.123	0.099	0.000
19	1	−1	1	−1	−1	1	1	1	−1	1	1	−1	−1	−1	−1	1	−1	−1	0.145	0.072	0.000	0.157	0.024	0.000
20	−1	1	−1	−1	−1	1	1	−1	1	1	1	−1	1	−1	−1	1	−1	−1	0.058	0.116	0.023	0.209	0.070	0.012
21	1	1	−1	−1	1	−1	1	1	−1	−1	1	1	1	−1	−1	1	−1	−1	0.039	0.066	0.013	0.145	0.039	0.013
22	−1	−1	1	1	−1	−1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	−1	0.012	0.174	0.000	0.070	0.023	0.000
23	1	1	1	1	−1	1	−1	1	1	1	−1	−1	1	1	−1	1	−1	−1	0.039	0.092	0.000	0.539	0.000	0.000
24	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	−1	1	−1	−1	0.131	0.131	0.012	0.179	0.167	0.012
25	−1	−1	−1	1	−1	−1	−1	−1	1	1	−1	−1	−1	−1	1	1	−1	−1	0.027	0.093	0.133	0.040	0.000	0.027

Table 8 (continued)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}	y_{CF_4}	y_{CF_5}
26	1	−1	−1	−1	−1	−1	−1	1	1	1	−1	1	1	−1	1	1	−1	−1	0.167	0.097	0.014	0.153	0.069	0.028
27	−1	1	1	1	−1	1	1	1	−1	−1	−1	−1	−1	1	1	1	−1	−1	0.060	0.217	0.024	0.301	0.012	0.012
28	1	1	−1	1	1	1	1	−1	1	−1	−1	1	−1	1	1	1	−1	−1	0.034	0.067	0.011	0.494	0.135	0.000
29	−1	−1	−1	−1	−1	1	−1	1	1	−1	1	−1	1	1	1	1	−1	−1	0.037	0.086	0.049	0.111	0.037	0.049
30	1	1	−1	1	1	1	−1	1	−1	−1	1	1	1	1	1	1	−1	−1	0.025	0.012	0.025	0.457	0.037	0.037
31	−1	1	−1	1	−1	1	−1	−1	−1	1	−1	−1	−1	−1	−1	−1	1	−1	0.057	0.086	0.186	0.186	0.000	0.000
32	1	−1	−1	−1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	1	−1	0.107	0.024	0.000	0.250	0.000	0.036
33	−1	−1	1	−1	−1	1	−1	−1	1	1	1	−1	1	−1	−1	−1	1	−1	0.066	0.197	0.026	0.145	0.026	0.026
34	1	1	−1	1	−1	1	−1	−1	−1	−1	1	1	1	−1	−1	−1	1	−1	0.000	0.036	0.048	0.530	0.000	0.012
35	1	−1	1	−1	−1	−1	1	−1	−1	1	−1	1	−1	1	−1	−1	1	−1	0.134	0.090	0.000	0.104	0.030	0.015
36	−1	1	1	−1	−1	1	1	1	−1	1	1	1	−1	1	−1	−1	1	−1	0.056	0.181	0.028	0.264	0.014	0.000
37	1	1	−1	−1	1	−1	1	1	1	−1	−1	−1	1	1	−1	−1	1	−1	0.067	0.045	0.000	0.236	0.034	0.045
38	1	−1	−1	1	−1	−1	1	1	1	−1	1	−1	−1	−1	1	−1	1	−1	0.206	0.059	0.044	0.059	0.029	0.044
39	−1	−1	−1	−1	1	1	−1	1	−1	1	1	1	−1	−1	1	−1	1	−1	0.012	0.073	0.037	0.110	0.000	0.061
40	1	1	1	1	1	1	1	1	−1	−1	1	−1	1	−1	1	−1	1	−1	0.016	0.066	0.033	0.590	0.033	0.049
41	−1	1	1	−1	1	−1	−1	1	1	−1	1	−1	−1	1	1	−1	1	−1	0.043	0.232	0.014	0.043	0.000	0.014
42	1	1	−1	−1	1	1	−1	1	1	1	1	1	−1	1	1	−1	1	−1	0.030	0.015	0.015	0.373	0.030	0.060
43	1	1	1	−1	−1	−1	−1	−1	1	1	−1	−1	1	1	1	−1	1	−1	0.048	0.119	0.012	0.202	0.012	0.071
44	−1	1	−1	1	−1	−1	1	−1	1	−1	1	−1	1	1	1	−1	1	−1	0.036	0.060	0.143	0.048	0.060	0.060
45	−1	−1	1	1	−1	−1	−1	1	1	−1	−1	1	1	1	1	−1	1	−1	0.024	0.241	0.012	0.048	0.036	0.036
46	1	−1	−1	1	−1	1	1	−1	1	1	1	1	1	1	1	−1	1	−1	0.090	0.034	0.079	0.281	0.191	0.067
47	−1	−1	1	−1	1	−1	1	−1	−1	−1	1	−1	−1	−1	−1	1	1	−1	0.012	0.256	0.012	0.035	0.012	0.012
48	1	1	−1	1	1	−1	−1	−1	1	1	1	−1	−1	−1	−1	1	1	−1	0.000	0.069	0.014	0.236	0.028	0.014
49	−1	1	1	−1	1	1	1	1	1	−1	−1	1	1	−1	−1	1	1	−1	0.100	0.171	0.014	0.229	0.129	0.029

Table 8 (continued)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}	y_{CF_4}	y_{CF_5}
50	1	−1	1	−1	−1	−1	−1	−1	1	−1	1	1	1	−1	−1	1	1	−1	0.099	0.154	0.000	0.055	0.055	0.033
51	1	1	1	−1	1	1	−1	1	−1	−1	−1	1	−1	1	−1	1	1	−1	0.026	0.052	0.000	0.429	0.013	0.013
52	−1	−1	−1	−1	−1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1	0.012	0.098	0.049	0.000	0.183	0.012
53	1	−1	−1	1	1	−1	1	1	−1	1	1	−1	1	1	−1	1	1	−1	0.139	0.028	0.042	0.056	0.056	0.014
54	−1	−1	−1	1	1	−1	−1	1	−1	−1	1	1	1	1	−1	1	1	−1	0.035	0.070	0.070	0.053	0.018	0.035
55	−1	1	1	1	1	1	−1	−1	1	1	1	1	1	1	−1	1	1	−1	0.036	0.327	0.073	0.255	0.036	0.000
56	1	1	−1	−1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	1	1	1	−1	0.053	0.092	0.039	0.447	0.013	0.053
57	−1	1	−1	1	1	−1	1	1	1	1	−1	−1	−1	−1	1	1	1	−1	0.056	0.122	0.111	0.067	0.033	0.011
58	1	1	1	1	−1	−1	−1	1	1	−1	1	1	−1	−1	1	1	1	−1	0.058	0.058	0.029	0.203	0.000	0.000
59	−1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	−1	1	1	1	−1	0.053	0.213	0.000	0.067	0.027	0.053
60	1	−1	−1	1	1	1	1	1	−1	−1	−1	1	1	−1	1	1	1	−1	0.143	0.032	0.032	0.127	0.079	0.048
61	1	−1	1	−1	1	−1	1	1	1	1	−1	−1	−1	1	1	1	1	−1	0.193	0.136	0.011	0.068	0.057	0.023
62	1	−1	1	1	1	1	−1	−1	1	−1	1	−1	−1	1	1	1	1	−1	0.146	0.134	0.024	0.195	0.012	0.037
63	1	1	−1	1	−1	−1	1	−1	−1	1	−1	−1	1	1	1	1	1	−1	0.021	0.021	0.032	0.232	0.042	0.084
64	−1	−1	1	−1	1	1	1	−1	−1	−1	−1	1	1	1	1	1	1	−1	0.027	0.243	0.027	0.135	0.054	0.068
65	−1	1	1	1	1	−1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	0.044	0.176	0.074	0.103	0.074	0.000
66	1	−1	1	−1	−1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	1	0.138	0.075	0.025	0.062	0.100	0.000
67	−1	−1	−1	−1	1	−1	−1	−1	1	−1	−1	1	1	−1	−1	−1	−1	1	0.041	0.068	0.014	0.027	0.110	0.000
68	1	1	−1	1	1	−1	−1	−1	1	−1	1	1	1	−1	−1	−1	−1	1	0.025	0.063	0.025	0.190	0.051	0.000
69	1	−1	−1	1	−1	1	−1	1	−1	1	1	1	1	−1	−1	−1	−1	1	0.148	0.057	0.045	0.239	0.023	0.011
70	−1	−1	−1	−1	1	−1	−1	1	1	1	1	−1	−1	1	−1	−1	−1	1	0.021	0.063	0.021	0.053	0.063	0.011
71	1	1	−1	−1	1	1	1	−1	−1	1	−1	1	1	1	−1	−1	−1	1	0.048	0.071	0.012	0.500	0.012	0.036
72	−1	1	−1	−1	1	1	1	−1	−1	−1	1	1	−1	−1	1	−1	−1	1	0.053	0.120	0.040	0.240	0.013	0.013
73	1	−1	−1	−1	1	1	−1	−1	−1	−1	1	−1	1	−1	1	−1	−1	1	0.113	0.070	0.014	0.239	0.014	0.028

Table 8 (continued)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}	y_{CF_4}	y_{CF_5}
74	−1	−1	1	−1	−1	−1	1	1	1	−1	1	1	1	−1	1	−1	−1	1	0.000	0.286	0.013	0.039	0.078	0.065
75	−1	−1	1	−1	−1	1	−1	−1	−1	−1	−1	−1	−1	1	1	−1	−1	1	0.000	0.262	0.000	0.175	0.000	0.000
76	1	−1	1	1	1	1	1	−1	−1	1	1	1	−1	1	1	−1	−1	1	0.132	0.132	0.000	0.132	0.053	0.013
77	1	1	−1	−1	−1	−1	1	1	1	−1	1	−1	1	1	1	−1	−1	1	0.070	0.058	0.047	0.233	0.023	0.012
78	−1	1	1	1	1	−1	1	1	−1	1	1	1	1	1	1	−1	−1	1	0.076	0.139	0.076	0.076	0.038	0.051
79	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	1	−1	1	0.000	0.120	0.040	0.480	0.013	0.000
80	−1	−1	−1	1	1	−1	1	−1	−1	1	−1	1	−1	−1	−1	1	−1	1	0.038	0.101	0.063	0.038	0.051	0.000
81	−1	−1	1	1	1	1	−1	−1	−1	−1	1	1	1	−1	−1	1	−1	1	0.025	0.262	0.125	0.188	0.000	0.000
82	1	−1	−1	−1	1	1	−1	−1	1	1	−1	1	−1	1	−1	1	−1	1	0.076	0.076	0.015	0.212	0.045	0.000
83	−1	1	−1	1	−1	1	1	1	1	−1	1	1	−1	1	−1	1	−1	1	0.088	0.038	0.138	0.200	0.112	0.012
84	1	1	−1	−1	−1	−1	−1	1	−1	1	1	1	−1	1	−1	1	−1	1	0.050	0.062	0.012	0.150	0.012	0.012
85	−1	1	1	−1	−1	−1	1	−1	−1	−1	−1	−1	1	1	−1	1	−1	1	0.081	0.243	0.000	0.041	0.000	0.000
86	−1	−1	1	−1	−1	1	−1	1	−1	1	−1	−1	1	1	−1	1	−1	1	0.027	0.240	0.000	0.200	0.000	0.000
87	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	−1	1	1	−1	1	−1	1	0.190	0.089	0.025	0.063	0.038	0.025
88	1	1	−1	1	1	1	1	1	1	1	1	−1	−1	−1	1	1	−1	1	0.038	0.013	0.026	0.436	0.090	0.038
89	1	−1	1	1	1	−1	−1	1	1	−1	1	1	−1	−1	1	1	−1	1	0.146	0.098	0.024	0.122	0.085	0.012
90	1	1	−1	−1	−1	1	1	−1	1	−1	−1	1	1	−1	1	1	−1	1	0.012	0.062	0.025	0.494	0.062	0.086
91	−1	1	−1	1	−1	−1	−1	1	1	1	−1	1	1	−1	1	1	−1	1	0.055	0.096	0.110	0.068	0.014	0.082
92	−1	1	1	−1	1	−1	−1	1	−1	1	−1	1	−1	1	1	1	−1	1	0.059	0.247	0.024	0.094	0.012	0.012
93	1	−1	1	1	−1	−1	1	1	−1	−1	−1	−1	1	1	1	1	−1	1	0.116	0.159	0.014	0.072	0.058	0.058
94	−1	−1	−1	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	−1	1	0.046	0.092	0.092	0.031	0.077	0.031
95	1	−1	1	1	1	−1	1	1	1	1	−1	1	−1	−1	−1	−1	1	1	0.101	0.159	0.014	0.087	0.101	0.029
96	1	1	1	−1	1	1	−1	1	1	1	1	−1	1	−1	−1	−1	1	1	0.013	0.215	0.000	0.506	0.013	0.038
97	−1	1	−1	−1	1	−1	1	−1	1	1	1	1	1	−1	−1	−1	1	1	0.090	0.064	0.026	0.115	0.090	0.013

Table 8 (continued)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}	y_{CF_4}	y_{CF_5}
98	1	1	1	1	− 1	1	1	− 1	1	1	1	− 1	− 1	1	− 1	− 1	1	1	0.022	0.112	0.000	0.494	0.079	0.011
99	− 1	− 1	− 1	1	− 1	1	− 1	− 1	1	− 1	1	1	− 1	1	− 1	− 1	1	1	0.013	0.101	0.038	0.152	0.051	0.063
100	− 1	− 1	1	1	1	− 1	− 1	− 1	− 1	1	− 1	− 1	1	1	− 1	− 1	1	1	0.062	0.225	0.050	0.100	0.012	0.012
101	− 1	1	1	− 1	1	1	− 1	1	− 1	− 1	1	− 1	1	1	− 1	− 1	1	1	0.068	0.311	0.000	0.230	0.014	0.014
102	1	− 1	1	− 1	− 1	1	− 1	1	1	− 1	− 1	1	1	1	− 1	− 1	1	1	0.148	0.136	0.000	0.235	0.062	0.025
103	− 1	− 1	− 1	− 1	− 1	− 1	1	1	− 1	1	− 1	1	1	1	− 1	− 1	1	1	0.013	0.039	0.078	0.026	0.026	0.000
104	1	1	− 1	1	− 1	− 1	1	1	− 1	− 1	− 1	1	− 1	− 1	1	− 1	1	1	0.027	0.014	0.108	0.149	0.014	0.000
105	1	1	1	− 1	1	− 1	− 1	− 1	1	− 1	− 1	1	− 1	− 1	1	− 1	1	1	0.059	0.074	0.000	0.206	0.103	0.029
106	− 1	1	1	1	− 1	1	1	− 1	− 1	1	− 1	− 1	1	− 1	1	− 1	1	1	0.086	0.157	0.000	0.229	0.000	0.086
107	1	− 1	− 1	− 1	− 1	1	1	− 1	1	1	− 1	− 1	1	− 1	1	− 1	1	1	0.103	0.038	0.013	0.192	0.115	0.038
108	− 1	− 1	− 1	1	1	1	− 1	1	1	1	− 1	− 1	1	− 1	1	− 1	1	1	0.000	0.126	0.138	0.126	0.011	0.069
109	1	1	− 1	1	1	− 1	− 1	1	− 1	1	− 1	− 1	− 1	1	1	− 1	1	1	0.023	0.070	0.000	0.186	0.000	0.023
110	− 1	1	− 1	− 1	− 1	1	− 1	− 1	1	1	− 1	1	− 1	1	1	− 1	1	1	0.034	0.080	0.046	0.115	0.023	0.011
111	1	− 1	1	− 1	1	− 1	− 1	1	− 1	− 1	1	− 1	1	1	1	− 1	1	1	0.229	0.100	0.000	0.071	0.014	0.057
112	− 1	1	− 1	− 1	1	1	1	1	− 1	1	1	− 1	− 1	− 1	− 1	1	1	1	0.048	0.024	0.024	0.217	0.024	0.000
113	1	− 1	− 1	− 1	1	− 1	1	− 1	− 1	− 1	1	1	− 1	− 1	− 1	1	1	1	0.200	0.012	0.000	0.106	0.059	0.000
114	1	− 1	− 1	1	1	1	− 1	1	1	− 1	− 1	− 1	1	− 1	− 1	1	1	1	0.065	0.078	0.026	0.221	0.065	0.039
115	− 1	− 1	1	1	− 1	1	1	1	1	1	1	− 1	1	− 1	− 1	1	1	1	0.000	0.169	0.000	0.205	0.120	0.072
116	1	1	1	1	− 1	− 1	− 1	1	− 1	1	− 1	1	1	− 1	− 1	1	1	1	0.027	0.110	0.014	0.178	0.014	0.000
117	1	− 1	− 1	1	− 1	1	1	− 1	− 1	− 1	− 1	− 1	− 1	1	− 1	1	1	1	0.068	0.055	0.014	0.247	0.041	0.027
118	− 1	1	1	1	1	− 1	1	− 1	− 1	− 1	− 1	1	− 1	1	− 1	1	1	1	0.082	0.301	0.082	0.041	0.041	0.000
119	− 1	1	− 1	1	− 1	1	− 1	1	1	− 1	− 1	− 1	1	1	− 1	1	1	1	0.057	0.092	0.126	0.172	0.011	0.000
120	− 1	− 1	− 1	1	− 1	− 1	− 1	1	− 1	− 1	− 1	− 1	− 1	− 1	1	1	1	1	0.000	0.092	0.211	0.053	0.000	0.066
121	− 1	1	1	1	− 1	1	− 1	− 1	1	− 1	1	− 1	− 1	− 1	1	1	1	1	0.041	0.151	0.041	0.151	0.068	0.041

Table 8 (continued)

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	*N*	*O*	*P*	*Q*	*R*	y_{LR}	y_{CF_1}	y_{CF_2}	y_{CF_3}	y_{CF_4}	y_{CF_5}
122	1	− 1	1	1	− 1	1	− 1	− 1	− 1	1	− 1	1	− 1	− 1	1	1	1	1	0.108	0.072	0.012	0.217	0.036	0.024
123	− 1	− 1	1	− 1	1	− 1	− 1	− 1	1	1	1	1	1	− 1	1	1	1	1	0.021	0.223	0.021	0.053	0.053	0.074
124	1	1	1	− 1	− 1	1	1	− 1	− 1	− 1	1	− 1	− 1	1	1	1	1	1	0.016	0.082	0.000	0.508	0.016	0.033
125	− 1	− 1	1	− 1	− 1	− 1	1	− 1	− 1	1	1	− 1	− 1	1	1	1	1	1	0.000	0.167	0.013	0.051	0.000	0.038
126	− 1	− 1	1	1	1	1	1	1	1	1	1	1	− 1	1	1	1	1	1	0.000	0.173	0.111	0.247	0.074	0.025
127	− 1	1	− 1	− 1	1	− 1	− 1	− 1	− 1	1	1	− 1	1	1	1	1	1	1	0.068	0.095	0.041	0.041	0.000	0.014
128	1	1	1	1	1	− 1	1	1	1	1	1	1	1	1	1	1	1	1	0.024	0.155	0.012	0.131	0.143	0.048

References

1. Addelman S (1963) Orthogonal main effect-plan for asymmetric factorial experiments. Technometrics 4(1):21–46
2. Ash T, Ginty M, Page R (2012) Landing page optimization: the definitive guide to testing and tuning for conversions. SYBEX Inc., Alameda
3. Bayne CK, Mitchell TJ (1978) D-optimal fractions of three-level factorial designs. Technometrics 20(4):369–80
4. Bien J, Taylor J, Tibshirani R (2013) A lasso for hierarchical interactions. Ann Stat 41(3):1111–41
5. Clifton B (2012) Advanced web metrics with Google analytics. Wiley, Indianapolis
6. Deng X, Hung Y, Lin CD (2015) Design for computer experiments with qualitative and quantitative factors. Stat Sin 25(4):1567–81
7. Joseph VR, Hung Y (2008) Orthogonal-maximin Latin hypercube designs. Stat Sin 18(1):171–86
8. Kang L, Joseph VR, Brenneman WA (2011) Design and modeling strategies for mixture-of-mixtures experiments. Technometrics 53(2):125–36
9. Kiefer J (1959) Optimum experimental designs. J R Stat Soc Ser B (Methodol) 21(2):272–319
10. Kiefer J, Wolfowitz J (1959) Optimum designs in regression problems. Ann Math Stat 30(2):271–94
11. King AB (2008) Website optimization. O'Reilly Media Inc., Sebastopol
12. Li W, Lin DKJ (2003) Optimal foldover plans for two-level fractional factorial designs. Technometrics 45(2):142–49
13. McKay MD, Beckman RJ, Conover WJ (1979) Comparison of three methods for selecting values of input variables in the analysis of output from a computer code. Technometrics 21(2):239–45
14. Montgomery DC, Runger GC (1996) Foldovers of 2^{k-p} resolution IV experimental designs. J Qual Technol 28(4):446–50
15. Montgomery DC (2012) Design and analysis of experiments, 8th edn. Wiley, New York
16. Ranjan P, Spencer N (2014) Space-filling hypercube designs based on randomization restrictions in factorial experiments. Stat Prob Lett 94:239–47
17. Scheffé H (1958) Experiments with mixtures. J R Stat Soc Ser B (Methodological) 20(2):344–60
18. Scheffé H (1963) The simplex-centroid design for experiments with mixtures. J R Stat Soc Ser B (Methodological) 25(2):235–63
19. Su H, Wu CFJ (2017) Funnel testing in webpage optimization: representation, design and analysis. Stat Theory Relat Fields 1(1):3–14
20. Tibshirani R (1996) Regression shrinkage and selection via the lasso. J Roy Stat Soc B Met 58(1):267–88
21. Wu CFJ, Hamada MS (2009) Experiments: planning, analysis, and optimization, 2nd edn. Wiley, Hoboken
22. Yang M, Biedermann S, Tang E (2013) On optimal designs for nonlinear models: a general and efficient algorithm. J Am Stat Assoc 108(504):1411–20
23. Zhao Y (2019) Statistical modeling and experimental design with contributions in environment, health care, and e-commerce. Doctoral dissertation. https://smartech.gatech.edu/handle/1853/61225. Accessed 15 Aug 2019